

Scope, Monotonicity, and Maximal Informativity *cannot be underestimated!*

A Compositional Analysis of an Apparent Linguistic Illusion

Ryan Walter Smith & Vera Hohaus (The University of Manchester)

Summary

Building on research on the semantics of comparison (Meier 2003, Beck & Rullmann 1999, Beck 2013) and the analysis of the **depth-charge illusion** (= *No head injury is too insignificant to be ignored*) in Hohaus & Bade (2022), we propose a compositional analysis of the **under-over illusion**. The alleged illusion arises when the **maximal informative degree** for the comparee in the comparison equals the **minimal degree**, rather than the maximal degree, a well-attested interpretative effect across degree constructions.

Prima facie, a type of inversion illusion:
Under = Over

“Logically, the literal meaning was the opposite of what I intended.”
(Barbara Wallraff, “Word Court”, *The Atlantic*, June 2004)

“This perplexing turn of phrase is extremely common, even among careful writers and speakers.”
(Ben Zimmer, “On Language”, *The New York Times*, 21st January 2011)

The Phenomenon

(1) *Chomsky’s importance cannot be underestimated.*
⇔ *Chomsky’s importance cannot be overestimated.*

Intended interpretation: ‘Chomsky is very important!’

Pedantic interpretation: ‘Chomsky is not important at all!’

The Analysis

A scope ambiguity, interacting with maximal informativity.

Surface scope CANNOT >> DegP: pedantic

Wide scope DegP >> CANNOT: intended

– Less-than comparison of the maximally informative degrees of two intensional degree sets

$$\llbracket \text{under-} \rrbracket = \lambda D'_{\langle d, \langle s, t \rangle \rangle} \cdot \lambda D_{\langle d, \langle s, t \rangle \rangle} \cdot \lambda w. \\ \text{the-MAX}_{\text{inf}}(D)(w) < \text{the-MAX}_{\text{inf}}(D')(w)$$

For any $p \in D_{\langle d, \langle s, t \rangle \rangle}$ and world w , $\text{the-MAX}_{\text{inf}}(p)(w) = \iota d [p(d)(w) = 1 \ \& \ \neg \exists d' [d \neq d' \ \& \ p(d')(w) = 1 \ \& \ [\text{if } p(d')(w) = 1, \text{ then } p(d)(w) = 1]]]$
(based on Beck 2013, p. 13, no. (36))

Maximal informativity is sensitive to entailment patterns, that is, the scalar properties of the degree set involved.

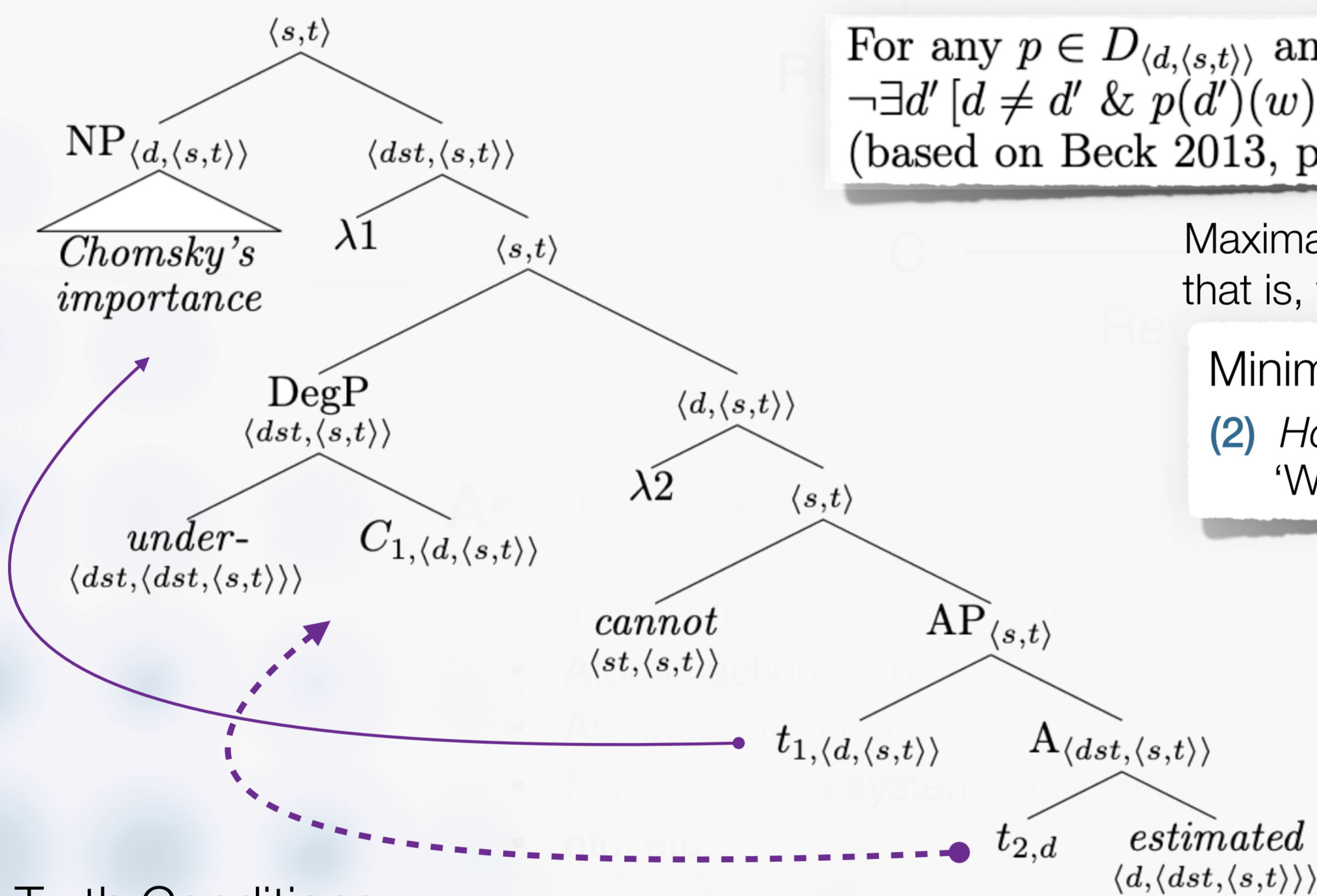
Minimal degree, maximally informative

(2) *How much arsenic is lethal in humans?*
‘What is the minimal amount that is lethal?’

– A modal semantics for the verbal root that maps a degree set to its contextually salient estimate:

$$\llbracket \text{estimate} \rrbracket = \text{simplified } \lambda d. \lambda D_{\langle d, \langle s, t \rangle \rangle} \cdot \lambda w. \\ \forall w' \in \text{ESTIMATE}(w) : D(d)(w') = 1$$

– A degree-based semantics for the noun phrase:
 $\lambda d. \lambda w. \text{IMPORTANCE}_w(C) \geq d$



Truth Conditions

Pedantic (with DegP >> CANNOT): “Always overestimated!”

$$\neg \exists w* [w* \text{ is accessible from } w_{\text{@}} \\ \& \text{the-MAX}_{\text{inf}}(\lambda d. \lambda w. d \in \text{ESTIMATE}_{w_{\text{@}}}(\lambda d'. \lambda w'. \text{IMPORTANCE}_{w'}(C) \geq d'))(w*) \\ < \text{the-MAX}_{\text{inf}}(\lambda d''. \lambda w''. \text{IMPORTANCE}_{w''}(C) \geq d'')(w*)]$$

Intended (with CANNOT >> DegP): “Very important!”

$$\text{the-MAX}_{\text{inf}}(\lambda d. \lambda w. \neg \exists w* [w* \text{ is accessible from } w \ \& \ d \in \text{ESTIMATE}_{w*}(\lambda d'. \lambda w'. \\ \text{IMPORTANCE}_{w'}(C) \geq d')])(w_{\text{@}}) < \text{the-MAX}_{\text{inf}}(\lambda d''. \lambda w''. \text{IMPORTANCE}_{w''}(C) \geq d'')(w_{\text{@}})$$

Nota bene: The first degree property in the comparison is upward-entailing!

‘There is no world where C’s estimated importance is less than his importance in the actual world.’

‘The maximally informative (= minimal) degree of estimated importance that C lacks in any world is less than his actual degree of importance.’

The minimum of the set of importance degrees that C lacks in any world is the highest degree that he has been assigned across worlds.

Discussion

- The claimed illusory but often intended reading arises **compositionally** from an interaction between the **scope** of negation, **monotonicity** and **maximal informativity**.
- These cases are highly reminiscent of the depth-charge illusion (= *No head injury is too trivial to ignore*).
- These illusions turn out to be closely related, and formal analysis can provide systematic explanations for both.

Kilburn Highest Factor Routine (amended)