Trivalent Exh and the Exclusion theory of summative predicates*

Mathieu Paillé • University of Calgary
SALT 33 • Yale • May 13, 2023

1 Introduction

- Predicates are SUMMATIVE if they are true of an individual by virtue of being true of that individual's parts, e.g.:

(1) The square is blue.
$\approx$ **All parts** of the square are blue

- This is opposed to INTEGRATIVE predicates that hold of individuals as atoms (e.g. *professor*, *happy*)

- This talk builds on previous work claiming that **the universal meaning in (1) is due to exhaustification**, and specifically **exclusion of related predicates** (other colour terms)

- I show that several shortcomings of this prior work can be overcome by replacing the standard bivalent Exh with its trivalent version 'Pexh' (Bassi et al. 2021):

(2) $$[\![\text{Pexh}_{\text{ALT}}(p)]\!] = \begin{cases} 1, \text{ if } [\![p]\!] = 1 \wedge \forall q \in \text{ALT}([\![q]\!] = 1 \rightarrow [\![p]\!] \subseteq [\![q]\!]); \\ 0, \text{ if } [\![p]\!] = 0; \\ \#, \text{ otherwise} \end{cases}$$

- I then show that this adoption of Pexh makes interesting predictions about predicates more generally (on the theory in Paillé 2022a, where both summative and integrative predicates involve the same kind of exhaustification) and are unexpectedly borne out.

*I would like to thank Bernhard Schwarz, Luis Alonso-Ovalle, Aron Hirsch, Nina Haslinger, anonymous reviewers for SALT, and audiences at Göttingen, Ottawa, and Calgary.

2 The Exclusion theory of summative predicates

- While summative predicates are true of *all* parts of their argument in positive sentences, they are true of *no* parts of their argument in negative sentences (Löbner 2000):

(3) a. The square is blue.
$\approx$ all of the square is blue
$\not\approx$ at least some of the square is blue
b. The square isn't blue.
$\not\approx$ not all of the square is blue
$\approx$ none of the square is blue

- Call this all-or-nothing quantification a HOMOGENEITY effect. How should it be captured?

- Harnish (1976), Levinson (1983), and Paillé (2022a) suggest that summative predicates are lexically existential:

(4) $[\![\mathsf{blue}]\!] = \lambda x.\exists y[y \sqsubseteq x \wedge \mathsf{blue}(y)]$. (abbreviated as '$\lambda x.\mathsf{blue}_{\exists}(x)$')

- (3b) follows immediately.

- For (3a), we suggest that **colours *exclude* other colour terms** through exhaustification:

(5) $[\![\text{Exh}_{\text{ALT}}(p)]\!] = 1$ iff $[\![p]\!] = 1$ and $\forall q \in \text{ALT}$ $([\![q]\!] = 1 \rightarrow [\![p]\!] \subseteq [\![q]\!])$

(6) a. Exh_{ALT} [The square is blue].
b. ALT $= \{$The square is blue$_{\exists}$, The square is white$_{\exists}$, The square is red$_{\exists}$, $\ldots\}$
c. $[\![(6a)]\!] = 1$ iff $\mathsf{blue}_{\exists}(s) \wedge \neg\mathsf{white}_{\exists}(s) \wedge \neg\mathsf{red}_{\exists}(s) \wedge \neg \ldots$

- If the square is at least partly blue and has no other colour, it must be entirely blue.[1]

- As it stands, this theory faces **three difficulties**, which can all be **overcome by adopting the trivalent Exh** of Bassi et al. (2021).

3 Problem 1: Truth-value gaps

- On the assumption that the falsity conditions of p are the same as the truth conditions of $\neg p$, summative predicates must have truth-value gaps (Löbner 2000; Križ 2015):

[1] To ensure that all parts must have a colour, we can add the adjective *clear* in the set of alternatives.

(7) $$[\![\text{The square is blue}]\!] = \begin{cases} 1, \text{ if the square is all blue;} \\ 0, \text{ if the square is not blue at all;} \\ \#, \text{ otherwise} \end{cases}$$

- Call this a HETEROGENEITY-GAP.
- As stated, the Exclusion account does not predict a heterogeneity-gap.
- I also show in Paillé 2022b that heterogeneity-gap does not arise from lexical meaning.[2]
  - ➽ The gap must arise in the composition.
- One way to get a gap is to replace the bivalent Exh of section 2 with the trivalent Pexh operator of Bassi et al. (2021), which **excludes alternatives in the truth conditions, but has falsity conditions based on only its prejacent:**

(8) $$[\![\text{Pexh}_{\text{ALT}}(p)]\!] = \begin{cases} 1, \text{ if } [\![p]\!] = 1 \wedge \forall q \in \text{ALT}([\![q]\!] = 1 \rightarrow [\![p]\!] \subseteq [\![q]\!]); \\ 0, \text{ if } [\![p]\!] = 0; \\ \#, \text{ otherwise} \end{cases}$$

- The Exclusion theory now obtains heterogeneity-gaps:

(9) $$[\![\text{Pexh}_{\text{ALT}} \text{ [the square is blue]}]\!] = \begin{cases} 1, \text{ if } \mathsf{blue}_\exists(s) \wedge \neg\mathsf{white}_\exists(s) \wedge \neg\mathsf{red}_\exists(s) \wedge \neg \ldots; \\ 0, \text{ if } \neg\mathsf{blue}_\exists(s); \\ \#, \text{ otherwise} \end{cases}$$

- Consider if the square is half blue, half white:
  - It is NOT the case that $\neg\mathsf{white}_\exists(s)$ holds, as needed for TRUTH.
  - It is also NOT the case that $\neg\mathsf{blue}_\exists(s)$ holds, as needed for FALSITY.

4 Problem 2: Non-maximality

- In plural predication, 'non-maximality' refers to the ability of positive sentences to tolerate exceptions (be non-universal)
- e.g., (10) can be felicitous even if not quite all the professors smiled (Križ 2015).

(10) The professors smiled.

- Non-maximality is observed with summative predicates too, e.g.:

(11) a. SCENARIO: *We are entering a bullfighting arena. Visitors are not permitted to wear any red, but my shirt is half red, half white. A security guard says:*
b. Your shirt is red, you can't enter the arena.

- With both plurals and summative predicates, non-maximality disappears with *all* (12), as do heterogeneity-gaps, suggesting a connection between the two (Križ 2015)

(12) a. All the professors smiled.
b. All of the shirt is red.

- There's nothing predicting non-maximality in the Exclusion theory as stated in section 2.
  - In fact, (11b) would negate that the shirt has any colour other than red.
- One way to obtain non-maximality is through Pexh together with Križ's theory of non-maximality, which **derives non-maximality from heterogeneity-gaps**.

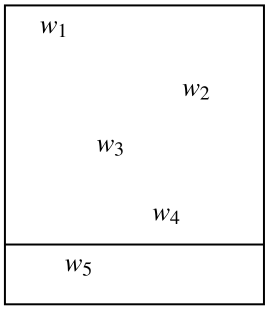
4.1 Križ's theory of non-maximality

- The first ingredient to Križ's (2015:76ff) theory is the standard assumption (e.g. van Rooij 2003) that a QUD partitions worlds by how they resolve it.
  - Let's say the QUD is 'How much red does the shirt have?'
  - A toy model:

w_1	Cell 1: The shirt is all red.
w_2	Cell 2: The shirt is mostly red.
w_3	Cell 3: The shirt is half red.
w_4	Cell 4: The shirt is mostly non-red.
w_5	Cell 5: The shirt is not red at all.

[2]Specifically, it does not arise due to a lexical 'all-or-nothing' presupposition à la Löbner 2000.

- Now let's say the QUD is 'Is the shirt red at all?' In this case, worlds w_1–w_4 are all in the same cell (they all answer 'yes'):



Cell 1: The shirt is at least partly red ('yes').

Cell 2: The shirt is not red at all ('no').

- From here, Križ (2015) suggests that **the maxim of Quality is weak**.
  - Rather than needing to say things that are true (Grice 1975), **speakers only need to say things that are 'TRUE ENOUGH.'**
  - For a sentence to be 'true enough,' it must identify a world IN THE SAME CELL as the real world.
  - **The sentence used cannot be false** in the real world—it can only be true or undefined.[3]
  - With Pexh rather than Exh, the Exclusion theory predicts that (13) would be undefined in worlds w_2–w_4 in our toy model.
  - If the real world is among w_2–w_4, then (13) can indeed be used as 'true enough' if they're in the same cell of the partition as w_1 (the world in which (13) is actually true).

(13) The shirt is red.

4.2 An alternative route to non-maximality (and why it doesn't work)

- An alternative way to get non-maximality would be to prune alternatives (cf. Bar-Lev 2021).[4]
  - It might not be necessary to adopt Pexh to generate truth-value gaps and use Križ's approach.

[3]Križ suggests a sentence cannot be used to address a QUD if a cell of the partition would contain worlds in which the sentence is true and worlds where it is false. Thus, QUD permitting, (10) can be used if only most professors smiled (being undefined), but not (12a) (being false).

[4]I thank an anonymous reviewer for bringing this up.

- For the Exclusion theory, (14) is what obtaining non-maximality by pruning alternatives would look like:

(14) a. (i) SCENARIO: *In the fall, you pick up a leaf that is mostly orange, but also partly green and brown.*
(ii) The leaf is orange.
b. ALT = {The leaf is orange$_\exists$, The leaf is pink$_\exists$, ~~The leaf is green$_\exists$~~, ~~The leaf is brown$_\exists$~~, The leaf is blue$_\exists$, ...}
c. $[\![\text{Exh}_{\text{ALT}}\ (14\text{a-ii})]\!] = 1$ iff the leaf is orange$_\exists$, maybe green$_\exists$ or brown$_\exists$, and no other colour

- On the Exclusion theory, the 'pruning' path to non-maximality differs from Križ's empirically:
  - The pruning approach can only create EXISTENTIAL force for colour terms.
  - (14c) means that the leaf has an orange part, and this part may be very small (the leaf might be mostly green/brown).
  - In contrast, Križ's approach can distinguish between many different quantificational forces, predicting that non-maximality can be stronger than mere existential meaning.
- Pruning cannot be all there is, since there are some clear cases with a 'more-than-existential' non-maximality:

(15) a. SCENARIO: *For a temporary art installation, you are making a large mosaic using leaves. There's a part of the drawing that should all be solid orange, but this part is still missing a lot of leaves. People will be looking at the mosaic from quite a distance to appreciate it as a drawing, so it's okay if the leaves you find are not actually fully orange.*
b. This leaf is orange.
✓ about a leaf that's mostly orange, but also a bit green/brown
✗ about a leaf that's mostly green/brown, but also a bit orange

➽ **(15) requires Križ's theory (and therefore Pexh!)**

- I leave open for now whether there are also cases where we really need to posit pruning.

5 Problem 3: Negation vs. other DE contexts

5.1 A locality constraint on Exh with predicates

- An odd fact about summative predicates (cf. Križ 2015 on plurals) is that, while they are weak under *not*, they remain strong in other DE contexts (see Paillé (2022a:ch. 2) for many more examples):

(16) If the square is blue, do five jumping-jacks. (≈ if the square is entirely blue, ...)

- On the Exclusion account, this means stipulating a locality constraint; Exh must appear below *if* in (16):

(17) ⟦If [Exh$_{\text{ALT}}$ the square is blue], do five jumping-jacks⟧
= 1 iff you should do five jumping-jacks if (the square is blue$_\exists$ $\wedge$ the square isn't any other colour)

- This locality constraint is not specific to DE contexts (Paillé 2022a:ch. 2/6)

- In (18), for instance, a (non-propositional) Exh must be below *exactly one* for *blue* to be strong.

(18) a. ⟦Exactly one [Exh$_{\text{ALT}}$ blue] square is high⟧

$$= 1 \text{ iff exactly one} \begin{pmatrix} \text{blue}_\exists\ \& \\ \text{not green}_\exists\ \& \\ \text{not red}_\exists \end{pmatrix} \text{square is high.}$$

b. *⟦Exh$_{\text{ALT}}$ [exactly one blue square is high]⟧

$$= 1 \text{ iff} \begin{cases} \text{exactly one blue}_\exists \text{ square is high } \wedge \\ \text{not exactly one green}_\exists \text{ square is high } \wedge \\ \text{not exactly one red}_\exists \text{ square is high} \end{cases}$$

- (18b) is a problem for two reasons:
  - it does not make *blue* mean 'all blue'
  - it creates non-intuited entailments about the amount of flags of other colours that are high

- In sum, Exh is generally obligatory and necessarily local with summative predicates.

5.2 No local Exh under *not*?

- Now observe that there is no Exh under *not* with summative predicates:[5]

(19) a. ⟦not [the square is blue]⟧ = 1 iff $\neg\mathsf{blue}_\exists(s)$. ✓
b. ⟦not [Exh$_{\text{ALT}}$ [the square is blue]]⟧ = 1 iff ¬(the square is only blue) ✗

- Whether or not this is surprising depends on how one characterizes the locality constraint on Exh.

- In Paillé 2022a, I claim that Exh must be within the predicate's XP, and suggest this could be due to an Agree relation where predicates probe for Exh.

- So Exh is definitely expected to surface below *not*, and in fact (19b) should really be (20):

(20) not [the square is [$_{a\text{P}}$ Exh$_{\text{ALT}}$ blue]]

- Why, then, is there no Exh observed below *not*?
  - We must derive an exception to this general locality constraint on the exhaustification of predicates.
  - This might not be impossible, but it's really not clear how to do it.

- With Pexh, the puzzle disappears:
  - ➽ The locality constraint can be claimed to be absolute, with Pexh vacuously[6] appearing below negation:

(21) ⟦not [Pexh$_{\text{ALT}}$ [the square is blue]]⟧

$$= \begin{cases} 1, \text{ if } \neg\mathsf{blue}_\exists(s); \\ 0, \text{ if } \mathsf{blue}_\exists(s) \wedge \neg\mathsf{red}_\exists(s) \wedge \neg\mathsf{white}_\exists(s) \wedge \neg \ldots; \\ \#, \text{ otherwise} \end{cases}$$

➽ Pexh seamlessly derives that negation behaves differently from other DE environments. This is a new kind of argument in favour of Pexh, not given by Bassi et al. (2021).

[5] It is probably not just *not*, but a small class of DE environments, namely NEGATIVE-FLAVOURED DE ENVIRONMENTS (under *not*, *no*, maybe *doubt*): Paillé 2022a:ch. 2 and ch. 6.

[6] Vacuously as far as truth conditions are concerned, but not general meaning: Pexh still affects the falsity/definedness conditions.

6 Interim summary before switching gears

- We've see three difficulties that the Exclusion theory of summative predicates (with bivalent Exh) faces:
  1. Truth-value gaps
  2. Non-maximality
  3. The difference between negation and other DE environments
- Adopting trivalent Pexh solves all these issues.
- At the same time, on the Exclusion theory in Paillé 2022a, adopting Pexh raises a big question for INTEGRATIVE predicates:

7 Do integrative predicates show truth-value gaps, too?

7.1 Motivating a unified composition for summative and integrative predicates

- We saw in the introduction that summative and integrative predicates differ in whether they involve part-quantification; it seems safe to assume they have a different lexical semantics:

(22) a. $[\![\text{green}]\!] = \lambda x.\exists y[y \sqsubseteq x \wedge \mathsf{green}(y)]$.
b. $[\![\text{chair}]\!] = \lambda x.\mathsf{chair}(x)$.

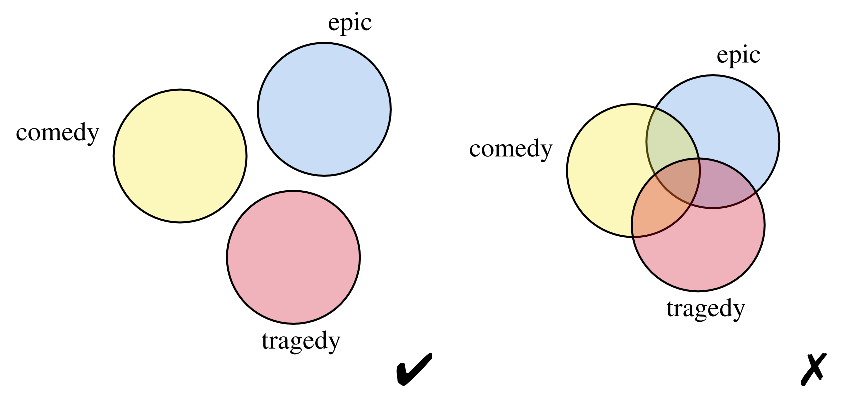
- Despite this difference in the LEXICAL semantics, they both involve strengthening to exclude other predicates from the same conceptual domain.
- We've already discussed that summative predicates are intuited as universal in basic sentences. One way to see this is from the contradiction that arises from 'co-predicating' such predicates:

(23) #The **white** flag is **green**.

- Co-predications make it easy to appreciate that **integrative predicates are strong too**—just not *quantificationally* (they don't quantify):

(24) a. #Some **comedies** are **tragedies**.
b. #This **fork** is a **spoon**.
c. #Some **federal** responsibilities are **provincial**.
d. #This **train** is a **plane**.

- This cannot be explained by world knowledge (cf. tragicomedies, hybrid vehicles, etc.)
- Integrative predicates are strong '**conceptually**' rather than quantificationally:



- This strength disappears in the same environments for both predicates, motivating a unified analysis of their intuited strength/weakness.

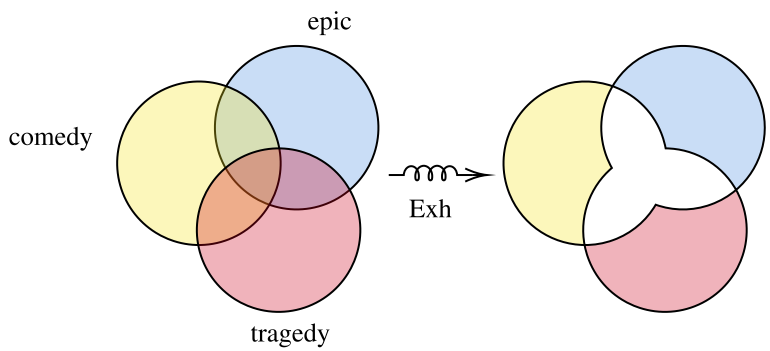
➽ They can all be co-predicated without a contradiction with *and* and *also*:

(25) a. The flag is both **white** and **green**. (summative, *and*)
b. The play is both a **comedy** and a **tragedy**. (integrative, *and*)

(26) a. The **white** flag is also **green**. (summative, *also*)
b. A tragicomedy is a **comedy** that is also a **tragedy**. (integrative, *also*)

- Therefore, in Paillé (2022a), I argue that **all** predicates to be **lexically weak** (e.g. *comedy* lexially includes tragicomedies), but undergo exhaustification (as already seen for summative predicates).
  - For integrative predicates, we end up with language acting as if it was cleaning up overlap in conceptual space:

(27) a. Exh_{ALT} [*Amphitryon* is a tragedy].
b. ALT $=$ {*Amphitryon* is a tragedy, *Amphitryon* is a comedy, *Amphitryon* is an epic, ...}
c. $[\![\text{(27a)}]\!] = 1$ iff $\mathsf{tragedy}(a) \wedge \neg\mathsf{comedy}(a) \wedge \neg\mathsf{epic}(a) \wedge \neg \ldots$



7.2 Truth-value gaps in integrative predication

- Integrative predicates have never been argued to have truth-value gaps/discourse-based weakness.[7]
- But in this talk, I adopted Pexh to create exactly those with summative predicates.
- If the semantic composition of integrative and summative predicates is fully unified (and assuming that language does not have both Exh and Pexh), (27) isn't quite right; we actually expect (28):

(28) $[\![\text{Pexh}_{\text{ALT}}\ [A \text{ is a tragedy}]]\!]$
$$= \begin{cases} 1, \text{if } \mathsf{tragedy}(a) \wedge \neg\mathsf{comedy}(a) \wedge \neg\mathsf{epic}(a) \wedge \neg \ldots; \\ 0, \text{if } \neg\mathsf{tragedy}(a); \\ \#, \text{otherwise} \end{cases}$$

➽ A truth-value gap is predicted when two same-domain predicates are true of an individual, but only one predicate is actually asserted.
  - Since *Amphitryon* is a tragicomedy, (28) would be undefined (it can't be true because *A* is a comedy, can't be false because *A* is a tragedy)
  - Call such a situation a 'failure to use two same-domain predicates' (FUTSDP).

- I now show the prediction is actually borne out: The theory correctly predicts never-described facts.
- I'll use the sentence (29), which is predicted to be undefined for a hybrid car-boat vehicle:

(29) This is a car.

[7] This doesn't mean there is never vagueness in what counts as an exemplar of a particular predicate/concept. For instance, what counts as a 'tree' could depend on whether you are writing a biology paper or engaging in everyday informal discourse. But this isn't a case of QUD-dependency like non-maximality.

7.2.1 Argument 1: *Well*-responses

- Križ (2015) suggests that *well*-responses are a marker of a sentence lacking a truth-value.
- He gives examples like (30) for truth-value gaps in plurals.

(30) A: The children are singing.
(1 if all singing, 0 if none singing, # otherwise)
B: {Well, ??No, #Yes}, half of them are.

- *Well* is *not* possible with sentences that are outright true (31a) or false (31b):

(31) a. A: *(about a car)* This is a car. (#*well* to a TRUE sentence)
B: #Well, it is indeed a car.
b. A: *(about a book)* This is a car. (#*well* to a FALSE sentence)
B: #Well, it's a book.

- So we predict that *well* should be acceptable as a response to a FUTSDP. It is:

(32) A: *(about a boat-car)* This is a car.
B: Well, it's a car that's also a boat.

➽ A's statement is neither true nor false.

7.2.2 Argument 2: Discourse-based weakness

- If there are truth-value gaps in FUTSDP, we expect Križ's mechanism for discourse-based weakness to kick in even with integrative predicates.
- Recall:
  - Križ (2015) argues that **undefined sentences are 'true enough'** if they are true in worlds in the same cell of the QUD as the real world.
- Indeed, for a car-boat hybrid, whether (33) can be used depends entirely on the QUD:

(33) This is a car.

(34) SCENARIO: *We are trying to sort various objects according to what they are.*
A: *(about a boat-car)* What kind of vehicle is this?
B: #It's a car.
C: Well, it's a car that's also a boat.

(35) SCENARIO: *A and B just robbed a bank and are looking for anything to drive away in.*
A: We need to find a car!
B: *(about a boat-car)* Here, this is a car!
A: Yes, great, let's go!

- That is, while the boat-car does not count as a car for the purposes of organizing artefacts (34), it does count as a car for the purposes of driving (35).

QUD: 'What kind of vehicle is this?'

w_1	Cell 1: 'This' is a car.
w_2	Cell 2: 'This' is a car-boat.
w_3	Cell 3: 'This' is a boat.

QUD: 'Can we drive this?'

w_1 w_2	Cell 1: Yes.
w_3	Cell 3: No.

- On Križ's theory, we can immediately capture this QUD-dependency if **there is a truth-value gap for (33)**.

7.3 Section summary

- On the assumption that both summative and integrative predicates involve obligatory exhaustification to exclude conceptually related predicates, we correctly predicted truth-value gaps for integrative predicates, despite these never having been described.
- If we describe truth-value gaps with summative predicates in terms of quantification ('*the square is blue* is undefined if some but not all parts of the square are blue'), it's not clear why one would expect something similar to hold for integrative predicates.
- But on the Exclusion theory, the truth-value gaps are both characterizable as FUTSDP: the speaker only uttered *blue/comedy* when they should have also uttered e.g. *white/tragedy*.

8 Conclusion

- We have seen three difficulties for the Exclusion theory of summative predicates:
  1. Heterogeneity-gaps
  2. Non-maximality
  3. Negation vs. other DE environments
- These shortcomings can all be overcome by replacing Exh with Pexh.
- On the theory of predicates in Paillé 2022a, we end up predicting truth-value gaps for integrative predicates too—which is correct, despite them never having been described as such.

References

Bar-Lev, Moshe E. 2021. An implicature account of Homogeneity and Non-maximality. *Linguistics and Philosophy* 44:1–53.

Bassi, Itai, Guillermo Del Pinal, & Uli Sauerland. 2021. Presuppositional exhaustification. *Semantics and Pragmatics* 14. Early access version.

Grice, H. Paul. 1975. Logic and conversation. In *Syntax and semantics*, ed. Cole Peter & Jerry L. Morgan, volume 3: Speech acts, 41–58. New York: Academic Press.

Harnish, Robert. 1976. Logical form and implicature. In *An integrated theory of linguistic ability*, ed. Thomas G. Bever, Jerrold J. Katz, & D. Terence Langendoen, 313–391. New York: Thomas Cromwell.

Križ, Manuel. 2015. Aspects of homogeneity in the semantics of natural language. Doctoral dissertation, University of Vienna, Vienna.

Levinson, Stephen C. 1983. *Pragmatics*. Cambridge: Cambridge University Press.

Löbner, Sebastian. 2000. Polarity in natural language: predication, quantification and negation in particular and characterizing sentences. *Linguistics and Philosophy* 23:213–308.

Paillé, Mathieu. 2022a. Strengthening predicates. Doctoral dissertation, McGill University, Montréal.

Paillé, Mathieu. 2022b. Subatomic homogeneity without the excluded middle presupposition: an argument from conjunction. In *Proceedings of Sinn und Bedeutung 26*, ed. Daniel Gutzmann & Sophie Repp, 658–676. Cologne: University of Cologne.

van Rooij, Robert. 2003. Questioning to resolve decision problems. *Linguistics and Philosophy* 26:727–763.