Contrary Negation as a combination of degree negation and a positive operator

Workshop: Diachronies of Negation — SALT 33

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# The Square of Opposition and Adjectives

Gotzner et al. (2018):

"It might be tempting to take Aristotle's square of opposition as a template to be applied to all kinds of Horn scales. However, it is particularly important in the context of adjectival scales that the meaning relations of the square of opposition do not generalise."
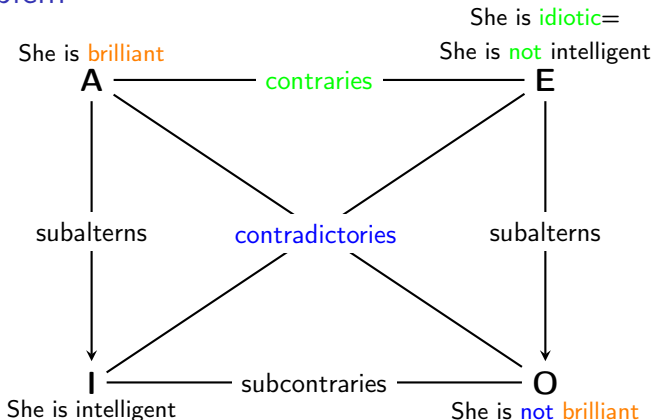
My Aim:

Show that they DO generalize.

The Problem
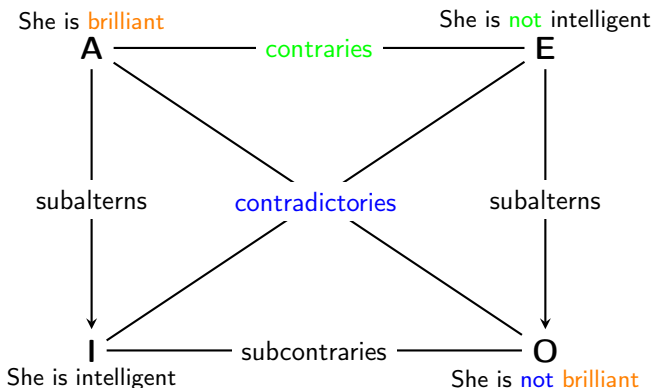
Gotzner et al. argue that **She is not intelligent** and **She is intelligent** are no contradiction in the system of meanings in combination with **brilliant**. How come?

The Problem

She is idiotic=
She is not intelligent

She is brilliant

A ———— contraries ———— E

subalterns   contradictories   subalterns

I ———— subcontraries ———— O

She is intelligent   She is not brilliant

Gotzner et al. argue that **She is not intelligent** and **She is intelligent** are no contradiction in the system of meanings in combination with **brilliant**. How come?

The Square hammered flat
A measurement scale for brilliance

|——————|——————|————|———-|

not intelligent not brilliant intelligent brilliant
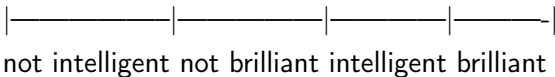
- The contraries occupy the poles of the measurement scale.
- The sequence **not intelligent** has the same meaning as **idiotic**. But there is a tenison between the negation used implicitly in **idiotic** and the negation **not**. Two **nots**?
- The contraries restrict the regions where their negations apply.
- and therefore there is gap between between being idiotic and being intelligent.
- The measurement scale and the square of opposition somehow do not fit — at first sight.

Questions

- Is there a **contrary negation** besides classical negation? Are there differences in meaning between sentential negation and affixal negation?

- Are there two types of Square of Opposition, one for the interaction between negation and regular quantifiers and one for adjectives and their antonyms?

- How do Horn Scales and the related implicatures (or other inferences) fit into the picture?

Literature and Hypothesis

- **Literature**: There is no consensus in the literature. Binary/Non-binary negation, Weak/strong negation (Jacobs, 1991), classical/contrary negation (Horn, 1989; Horn and Wansing, 2020). Heim (2008),von Stechow (2009) posit a difference in scope in combination with a degree quantifier (see also DeClerq and Wyngared, 2017) .

- **Basic insight**: The problems for non-classical negation involve gradable predicates.

- **Analysis**: Seemingly non-binary negation can be decomposed into a (hidden) universal quantifier over degrees (the positive operator) and classical logical negation. The negation hypothesis remains valid. But negation is a type-flexible operator.

Map of the talk

- **Data**: Problematic cases are cases with gradable adjectives.
- **Theory**: Adopt a semantics for gradable adjectives, positive operator (universal quantifier) and a type-flexible negation operator.
- **Application**: differences in meaning are due to differnces in scope of classical negation with the positive operator: another scope effect.
- **Bigger picture 1**: Squares of opposition: Quantification and Predication with gradable adjectives are interrelated by negation.
- **Bigger picture 2**: Pragmatic inferences: Scalar implicatures, Negative Strengthening and Scale Reversal (Gotzner et al., 2018).

Affixal negation is resticted in scope
Scope effects: universal quantifier (Jacobs, 1991)

(1) **Alle Politiker sind nicht verheiratet.**
'Every politician is not married' ambiguous

(2) **Alle Politiker sind unverheiratet.**
'Every politician is unmarried' not ambiguous

Sentence (1) is ambiguous. Sentence (2) is not ambiguous. The
scope of affixal negation is limited to the predicate **verheiratet**
'married'. Sentential negation interacts with quantifiers and allows
for scope ambiguities.

Affixal negation may not cancel presuppositions
Scope effects: Presupposition triggers (Horn, 1989)

(3) **Der König von Frankreich ist nicht verheiratet. Es gibt gar keinen König von Frankreich.**
'The king of France is not married. There is no king France.'

(4) **Der König von Frankreich ist unverheiratet. #Es gibt gar keinen König von Frankreich.**
'The king of France is unmarried. #There is no king of France.'

Sentence (3) allows for cancellation of the presupposition triggered by the definite description. Sentence (4) does not. The scope of affixal negation is limited to the predicate **verheiratet** 'married'. Sentential negation interacts with definite descriptions and allows for scope ambiguities.

Solution: Negation is type-flexible
Analogous argument for conjunction in Partee and Roth (1983)

- $[\![\mathbf{nicht}_1]\!]^s = \lambda p.1 - p$ sentential negation
- $[\![\mathbf{nicht}_2]\!]^s = \lambda P^{et}.\lambda x.[\![\mathbf{nicht}_1]\!]^s(P(x))$ predicate negation

- $[\![\mathbf{un}\text{-}]\!]^s = \lambda P^{et}.\lambda x.[\![\mathbf{nicht}_1]\!]^s(P(x))$ affixal negation

- Semantic argument: narrow scope of **un-** wrt. overt quantifiers.
- Semantic/pragmatic argument: Presupposition cancellation is impossible with **un-**.
- Morphological argument: **un-** is a bound morpheme. Its application is limited to predicates.

Narrow scope negation:
predicate negation $=$ affixal negation

$$\llbracket\textbf{Alle Politiker sind unverheiratet}\rrbracket^{s^*}$$
$$= \vdash\text{All politicians are unmarried in s*}\dashv$$

$\llbracket\textbf{Alle Politiker}\rrbracket^{s^*}$
$$|$$
$$=\lambda X.\vdash Pol_s \subseteq\downarrow X\dashv$$

$\llbracket\textbf{unverheiratet}\rrbracket^{s^*}$
$$=\lambda x.\llbracket\textbf{nicht}_1\rrbracket^{s^*}(\lambda x.\vdash x \text{ is married in } s^*\dashv(x))$$
$$=\lambda x.1- \vdash x \text{ is married in } s^*\dashv$$

$\llbracket\textbf{un-}\rrbracket^{s^*}$
$$=\lambda P^{et}.\lambda x.\llbracket\textbf{nicht}_1\rrbracket^{s^*}(P(x))$$

$\llbracket\textbf{verheiratet}\rrbracket^{s^*}$
$$= \lambda x.\vdash x \text{ is married in s*}\dashv$$

The Negation Hypothesis
Paraphrasing Jacobs 1991, 569

For every natural language *L* the following holds:

In an adequate semantic theory of *L*, every negative expression is
represented (even if only partly) with classical logical negation.

Adequate semantic theory for German: method of direct
interpretation as introduced in Zimmermann (2016).

Exceptions to the Neg Hypothesis
Allegedly no scope effect, still difference in meaning: Jacobs, 593

(5)   **Der König von Frankreich erwies sich als ungebildet.**
      'The king of France proved to be uneducated.'   strong
      negation

(6)   **Der König von Frankreich erwies sich als nicht
      gebildet.**
      'The kind of France proved not to be educated.'   weak
      negation

$\Rightarrow \llbracket \textbf{un-}_2 \rrbracket^s \neq \lambda P^{et}.\lambda x. \llbracket \textbf{nicht}_1 \rrbracket^s (P(x))$?

There is difference in meaning. The point: sentential negation does not interact scopally with the definite description. The **als**-phrase limits the scope of negation to the predicate **gebildet** 'educated', Jacobs (1991).

Interpretation of gradable adjectives
Following von Stechow (2009)

- The interpretation of gradable adjectives is associated with points on a scale (Cresswell, 1976).
- The points on the scale are called degrees and values of a measure function. i.e. numbers.
- **gebildet** 'educated' is interpreted with respect to a measure function: $MEASURE_{EDU}$. Some other adjectives may come with measure units: **long**, Length measured in meters: $MEASURE_{LENGHT,m}$.
- Adjectives express relations between an individual from a set of individuals A and a degree from the set of reels $\mathbb{R}$.

- $[\![\textbf{gebildet}]\!]^s = \lambda d.\lambda x.\vdash MEASURE_{EDU}(s)(x) \geq d \dashv$

The Positive, von Stechow (2009)

- The positive form of adjectives is complex, though. I adopt the view that there is an (invisible) positive morpheme that relates two sets of degrees. The second set is a contextually determined interval somewhere in the middle of the scale: delineation interval. That is: The positive morpheme corresponds to a universal quantifier.

  Definition Positive

- $[\![\textbf{POS}]\!]^{s,c} = \lambda D_1 . \vdash NORM_s^c \subseteq \downarrow D_1 \dashv$

- $[\![\textbf{DKF ist POS-gebildet}]\!]^{s,c} = 1$ iff
  $\forall d[d \in NORM_s^c \rightarrow \text{MEASURE}_{\text{EDU}}(s^*)(\text{TKF}) \geq d]$

Affixal Negation is type-flexible as well, von Stechow (2009)

- $[\![\textbf{gebildet}]\!]^s = \lambda d.\lambda x. \vdash\text{MEASURE}_{\text{EDU}}(s)(x) \geq d\dashv$

- $[\![\textbf{un-}_2]\!]^s = \lambda R^{d,et}.\lambda x.[\![\textbf{nicht}_1]\!]^s(R(d)(x))$ degree negation

- $[\![\textbf{un}_2\textbf{-gebildet}]\!]^s$
  $= \lambda d.\lambda x.1 - \vdash\text{MEASURE}_{\text{EDU}}(s)(x) \geq d\dashv$
  $= \lambda d.\lambda x. \vdash\text{MEASURE}_{\text{EDU}}(s)(x) < d\dashv$

- The Negation Hypothesis is still met.

Strong Negation: narrow scope wrt the positive operator
Negation negates the comparison relation: internal negation/degree negation

- $[\![\textbf{DKF ist POS-gebildet}]\!]^{s,c} = 1$ iff
  $\forall d[d \in NORM_s^c \rightarrow \text{MEASURE}_{\text{EDU}}(s^*)(\text{TKF}) \geq d]$

- $[\![\textbf{DKF ist POS-[[un-gebildet]]}]\!]^{s,c} = 1$ iff
  $\forall d[d \in NORM_s^c \rightarrow NOT : \text{MEASURE}_{\text{EDU}}(s^*)(\text{TKF}) \geq d]$
  iff $\forall d[d \in NORM_s^c \rightarrow \text{MEASURE}_{\text{EDU}}(s^*)(\text{TKF}) < d]$
  relation flips

- Strong negation is a name for narrow scope negation with
  respect to a degree quantifier.

Strong Negation: narrow scope wrt the positive operator

Negation negates the comparison relation: internal negation/degree negation

- $\llbracket\textbf{DKF ist POS-gebildet}\rrbracket^{s,c} = 1$ iff
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  iff $\forall d[d \in NORM_s^c \rightarrow \text{MEASURE}_{\text{EDU}}(s^*)(\text{TKF}) < d]$
  relation flips

- Strong negation is a name for narrow scope negation with respect to a degree quantifier.

Weak negation: wide scope wrt the positive operator

Negation negates the positive operator (universal quantifier): external negation

- $[\![\text{DKF ist POS-gebildet}]\!]^{s,c} = 1$ iff
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- $[\![\text{DKF ist nicht POS-gebildet}]\!]^{s,c} = 1$ iff
  $\text{NOT} : \forall d[d \in NORM_s^c: \text{MEASURE}_{\text{EDU}}(s^*)(\text{TKF}) \geq d]$ iff
  $\exists d[d \in NORM_s^c: \text{MEASURE}_{\text{EDU}}(s^*)(\text{TKF}) < d]$
  quantifier and relation flips

- Weak negation is a name for wide scope negation with respect
  to a degree quantifier.

Weak negation: wide scope wrt the positive operator

Negation negates the positive operator (universal quantifier): external negation

- $[\![$**DKF ist POS-gebildet**$]\!]^{s,c} = 1$ iff
  $\forall d[d \in NORM_s^c \rightarrow \text{MEASURE}_{\text{EDU}}(s^*)(\text{TKF}) \geq d]$

- $[\![$**DKF ist nicht POS-gebildet**$]\!]^{s,c} = 1$ iff
  $\text{NOT} : \forall d[d \in NORM_s^c : \text{MEASURE}_{\text{EDU}}(s^*)(\text{TKF}) \geq d]$ iff
  $\exists d[d \in NORM_s^c : \text{MEASURE}_{\text{EDU}}(s^*)(\text{TKF}) < d]$
  quantifier and relation flips

- Weak negation is a name for wide scope negation with respect to a degree quantifier.

Dual Negation: external and internal negation combined
The positive operator (universal quantifier) intervenes between the two negations

- $[\![$**DKF ist POS-gebildet**$]\!]^{s,c} = 1$ iff for all degrees
  $d \in NORM_s^c \rightarrow \text{MEASURE}_{\text{EDU}}(s^*)(\text{TKF}) \geq d$

- $[\![$**DKF ist nicht [POS-[un-gebildet]]**$]\!]^{s,c} = 1$ iff NOT : d :
  $\forall d[d \in NORM_s^c \rightarrow \text{NOT} : \text{MEASURE}_{\text{EDU}}(s^*)(\text{TKF}) \geq d]$iff
  $\exists d[d \in NORM_s^c : \text{MEASURE}_{\text{EDU}}(s^*)(\text{TKF}) \geq d]$
  only quantifier flips

- Dual negation of a universal quantifier corresponds to
  existential quantification (An affirmative is expressed by the
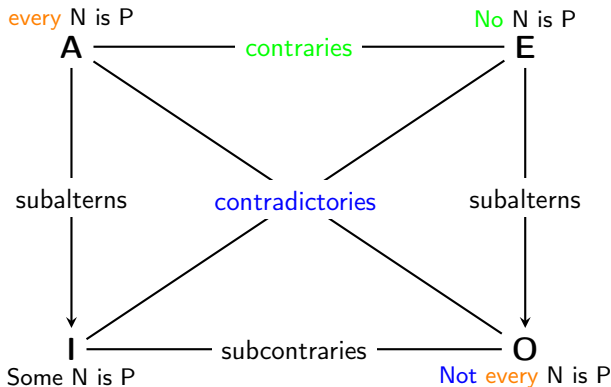 negation of the contrary: litotes)

Dual Negation: external and internal negation combined

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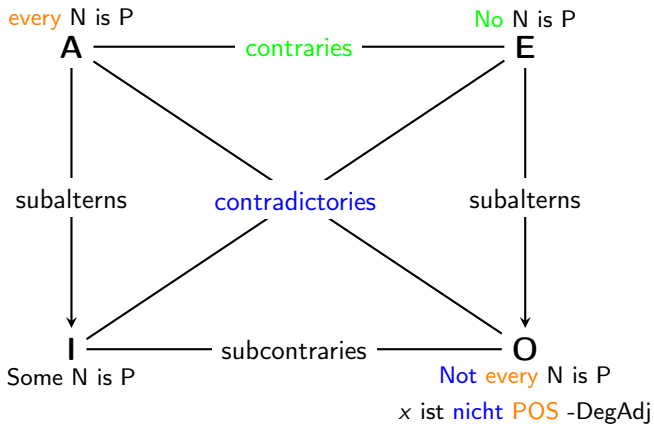 negation of the contrary: litotes)

The Square of Opposition: Quantification in general

Aristotle, Horn (1989): Relations between Quantifications

The Square of Opposition: Quantification in general

Aristotle, Horn (1989): Relations between Quantifications

every N is P   No N is P
A —————— contraries —————— **E**

subalterns   contradictories   subalterns

I —————— subcontraries —————— **O**

Some N is P   Not every N is P

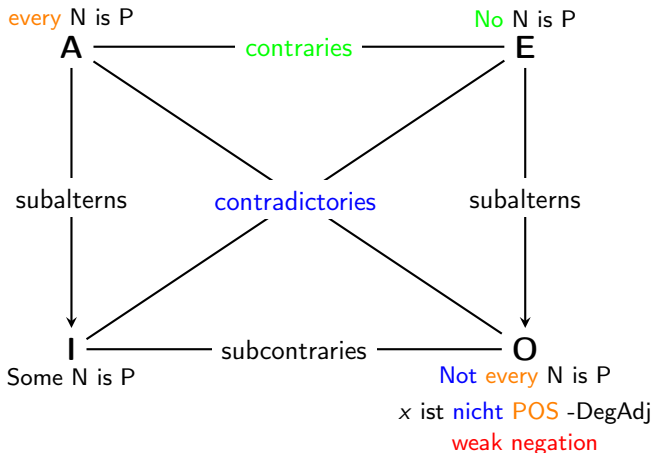x ist nicht POS -DegAdj

The Square of Opposition: Quantification in general

Aristotle, Horn (1989): Relations between Quantifications

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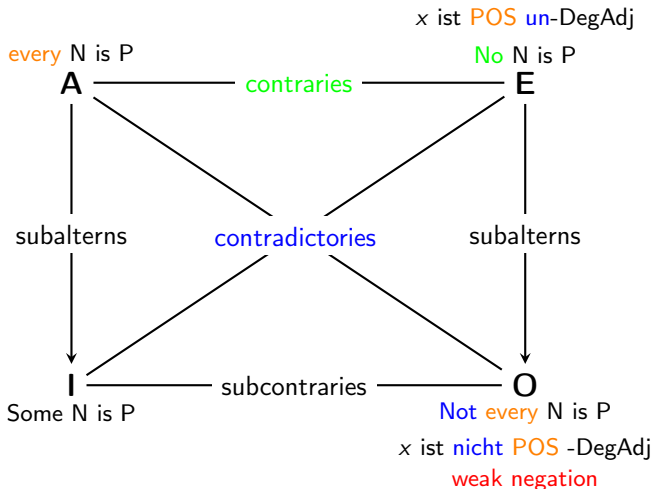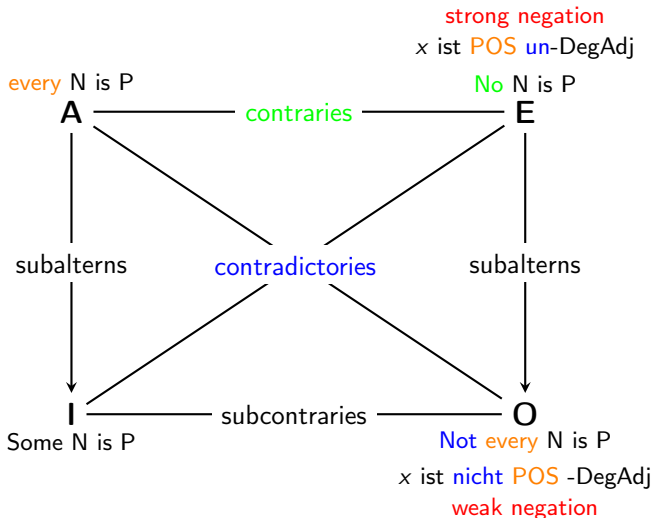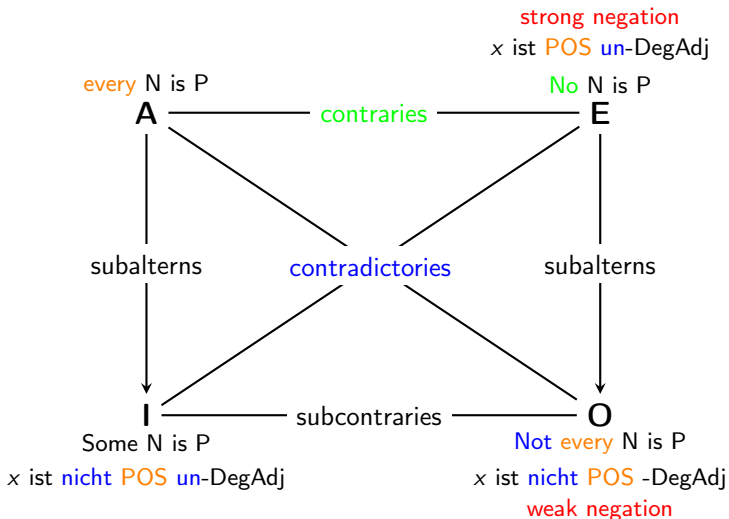
The Square of Opposition: Quantification in general

Aristotle, Horn (1989): Relations between Quantifications

The Square of Opposition: Quantification in general

Aristotle, Horn (1989): Relations between Quantifications

strong negation
x ist POS un-DegAdj

every N is P
A ———— contraries ———— E   No N is P

subalterns   contradictories   subalterns

I ———— subcontraries ———— O

Some N is P   Not every N is P
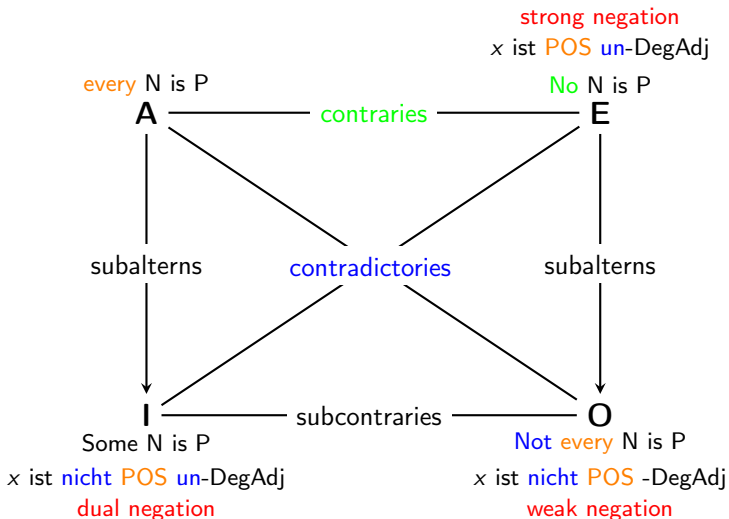x ist nicht POS un-DegAdj x ist nicht POS -DegAdj
weak negation

The Square of Opposition: Quantification in general

Aristotle, Horn (1989): Relations between Quantifications

The Square of Opposition: Modals: 2 ways

Aristotle, Horn (1989): Relations between Quantifications

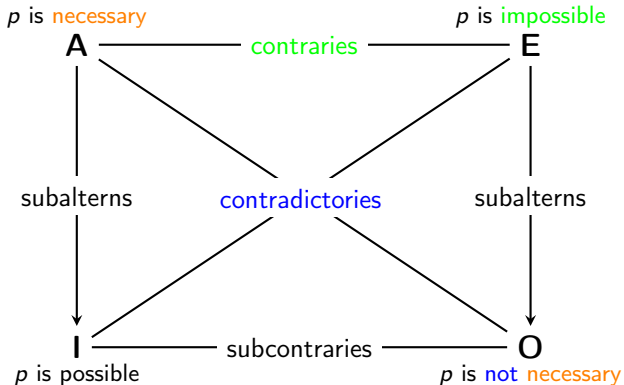## The Square of Opposition: Modals: 2 ways
Aristotle, Horn (1989): Relations between Quantifications

The Square of Opposition: Modals: 2 ways

Aristotle, Horn (1989): Relations between Quantifications

strong negation

x is POS im-possible $\neq$

The Square of Opposition: Modals: 2 ways

Aristotle, Horn (1989): Relations between Quantifications

x is POS possible

p is necessary

strong negation

x is POS im-possible $\neq$

p is impossible

A ——— contraries ——— E

subalterns   contradictories   subalterns

I ——— subcontraries ——— O

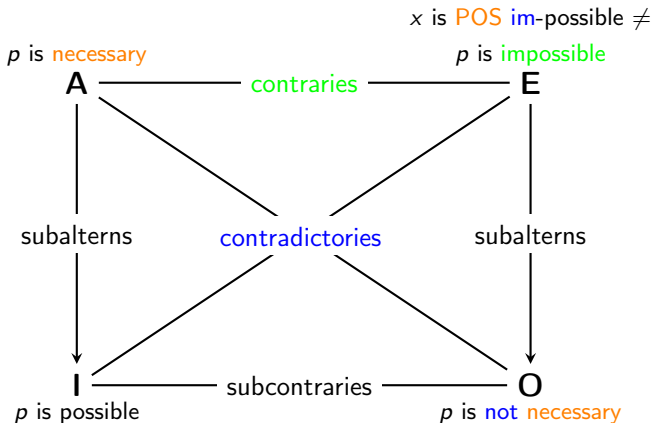p is possible
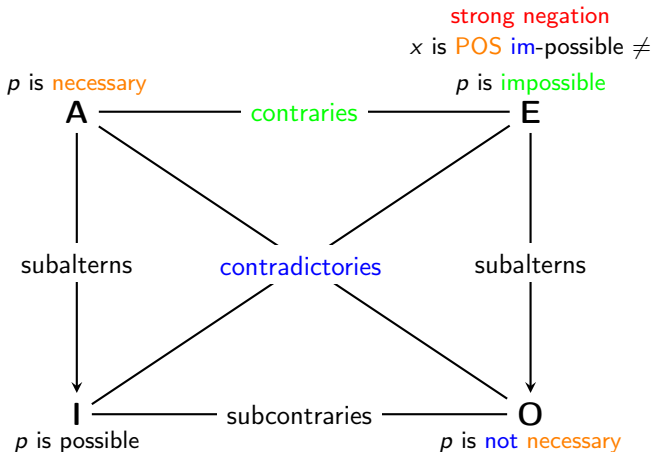
p is not necessary
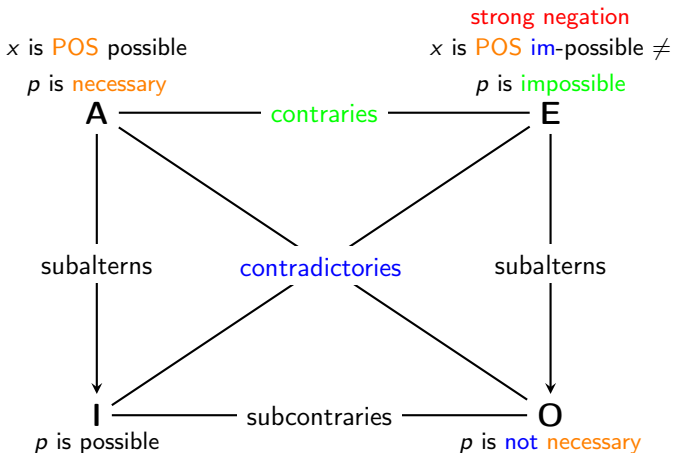
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happy, **unhappy** and **not happy**
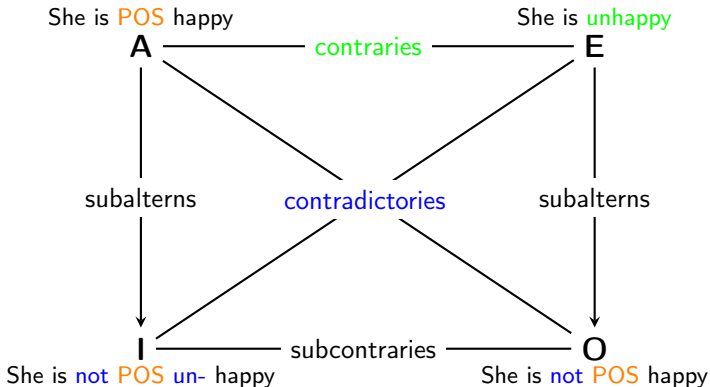
happy, **unhappy** and **not happy**

brilliant, **intelligent** and **idiotic**

She is POS brilliant
A ——————— contraries ——————— E She is POS un-brilliant

subalterns   contradictories   subalterns

I ——————— subcontraries ——————— O
She is not POS un-brilliant   She is not POS brilliant
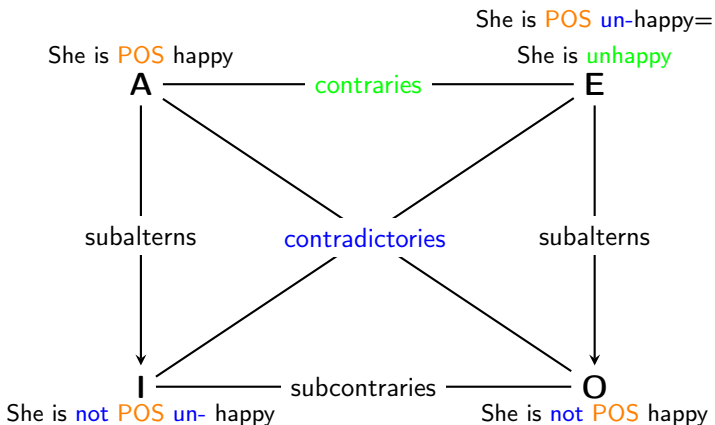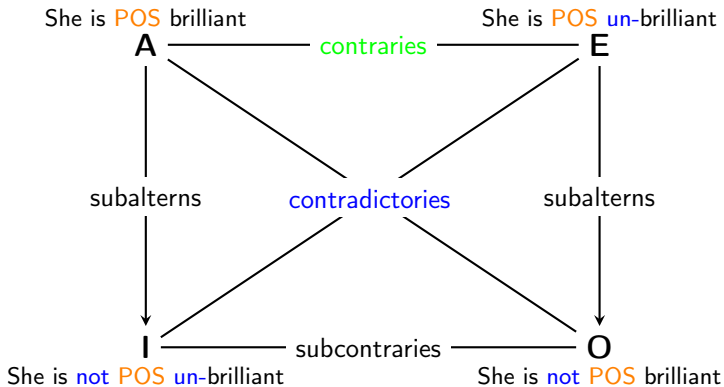
brilliant, **intelligent** and **idiotic**

brilliant, **intelligent** and **idiotic**

She is not POS intelligent =
She is not intelligent =

She is POS brilliant   She is POS un-brilliant

A ———————— contraries ———————— E

subalterns   contradictories   subalterns

I ——————— subcontraries ——————— O

She is not POS un-brilliant   She is not POS brilliant
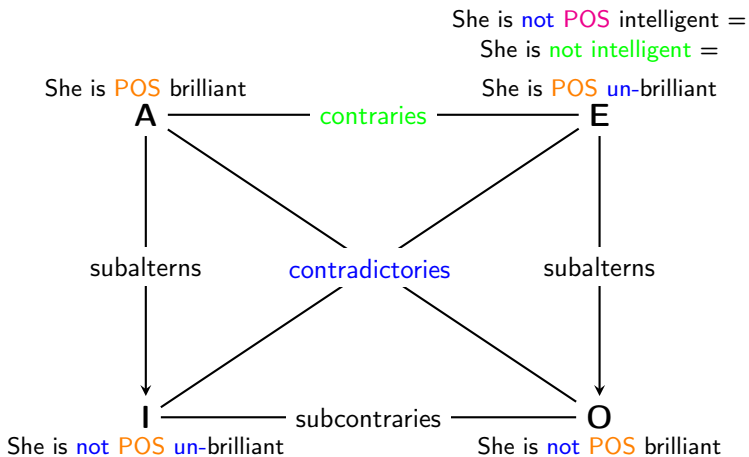
brilliant, **intelligent** and **idiotic**
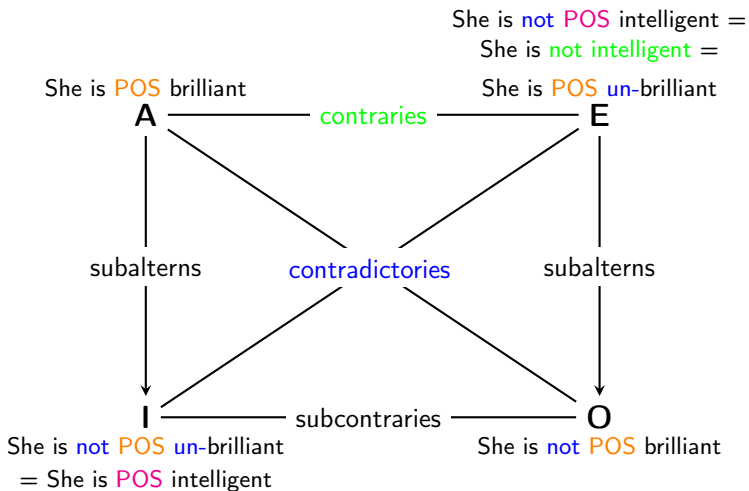
Existential Degree Quantification

Definition Positive as an existential operator

- $[\![\textbf{POS}_2\text{-}]\!]^{s,c} = \lambda D_1.\vdash NORM_s^c \cap \downarrow D_1 \neq \emptyset \dashv$

- $[\![\textbf{She is POS}_2\text{-intelligent}]\!]^{s,c} = 1$ iff
  $\exists d[d \in NORM_s^c \ \& \ \text{MEASURE}_{\text{INTELL}}(s^*)(\text{she}) \geq d]$

- **intelligent** patterns with **possible** in one reading and **some**.

Intermediate Summary

- Logical relations between (un)negated quantified statements
  may be visualized by the square of oppositions.
- One and the same adjective may participate in different
  entailment scales, though.
- There is no need for contrary negation: It is a scope issue and
  an issue of quantificational force.
- Proposal: The degree operator may shift from universal to
  existential meaning with different scope properties with respect
  to negation.
- The Square of Opposition does generalize to adjectival
  meanings.
- Two negations do not cancel out because a POS-Operator
  may intervene.

Bigger Picture: 3 Types of Inferences

Gotzner et al. (2018) investigated pairs of adjectives with respect to inferences
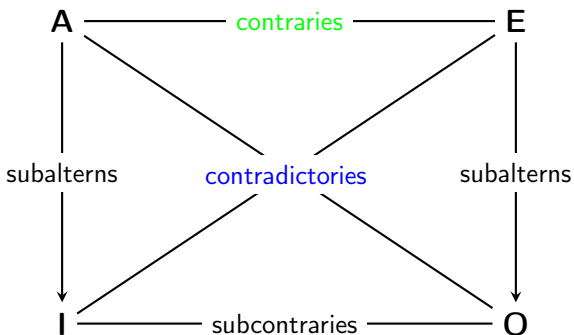
- Scalar implicature, positive: I $\rightarrow$ O   SI-pos
- Scalar implicature, negative: O $\rightarrow$ I   SI-neg
- Negative strengthening: O $\rightarrow$ E   NegS
- Experimental findings: SI-pos and SI-neg correlate, NegS and SI anti-correlate, for particular pairs of antonyms.

Scalar implicature, positive (SI-pos)

(7)   **She is not idiotic** (I) $\hookrightarrow$ **She is not intelligent** (O)

```
idiotic                    intelligent
--------------/-----/---------------------→
              xxxxxxxxxxxxxxxxxxxxxxx not idiotic (semantics)
xxxxxxxxxxxxxx                        not intelligent (pragmatics)
      xxxxx                           Overall meaning
```

- Only 2 alternatives are matched with the measurement scale
- $\langle$**intelligent**$_A$, **not idiotic**$_I$, **idiotic**$_E$, **not intelligent**$_O\rangle$
  (negation of the stronger proposition A)

Scalar implicature, positive (SI-pos)
medium size strengthening

(8)   **She is intelligent** (I) $\hookrightarrow$ **She is not brilliant** (O)
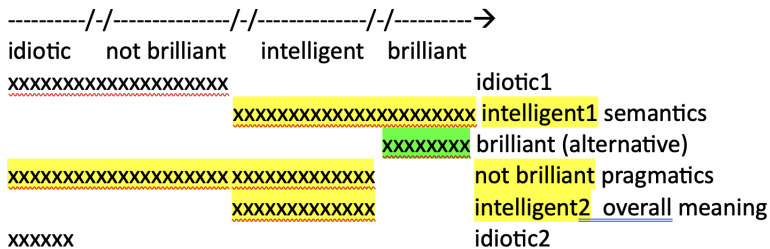
```
----------/-/---------------/-/--------------/-/----------→
idiotic    not brilliant    intelligent   brilliant
xxxxxxxxxxxxxxxxxxxxx                                 idiotic1
                 xxxxxxxxxxxxxxxxxxxxxxx intelligent1 semantics
                                 xxxxxxx brilliant (alternative)
xxxxxxxxxxxxxxxxxxxxx xxxxxxxxxxxxx      not brilliant pragmatics
                 xxxxxxxxxxxxx           intelligent2 overall meaning
xxxxxx                                   idiotic2
```

- 4 alternatives are matched with the measurement scale
- $\langle$ **brilliant**$_A$, **intelligent**$_I$, **idiotic**$_E$, **not brilliant**$_O\rangle$

Scalar implicature, negative (SI-neg)
medium size strengthening

(9)   **She is not brilliant** (O) ↪ **She is not idiotic** (I)

```
----------/-/---------------/-/--------------/-/---------->
idiotic    not brilliant    intelligent   brilliant
xxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxx                         not brilliant1 (semantics)
xxxxxxxx                                                      idiotic (alternative)
          xxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxx             not idiotic1 (pragmatics)
          xxxxxxxxxxxxxxxxxxxxxxxxxxxxx                        overall meaning
          xxxxxxxxxxxx                                         not brilliant2
```
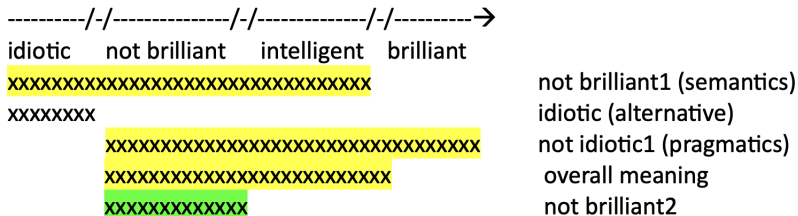
- 4 alternatives are matched with the measurement scale
- ⟨**brilliant**$_A$, **not idiotic**$_I$, **not intelligent**$_E$, **not brilliant**$_O$⟩

Negative Strengthening (NegS)
medium size elimination possible

(10)   He is not happy (O) ↪ He is unhappy (E)

```
Unhappy                  happy
------------------//--------------------→
xxxxxxxxxxxxxxx                  unhappy
xxxxxxxxxxxxxxx                  not happy
```
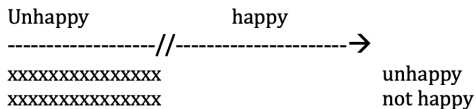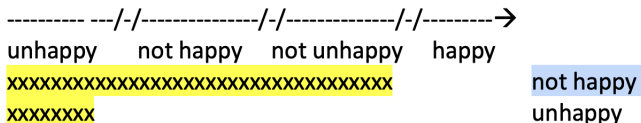
- Horn's explanation: this is an instance of the inference pattern Modus Tollendo Ponens. It involves the law of excluded middle. the gap shrinks to a point: contradiction
- The Square of opposition collapses
- 2 alternatives are matched with the measurement scale
- $\langle$happy$_A$, not unhappy$_I$, unhappy$_E$, not happy$_O\rangle$

Negative Strengthening (NegS)
medium size elimination not possible

(11)   **He is not happy** (O) *↪ **He is unhappy** (E)

```
---------- ---/-/---------------/-/--------------/-/---------→
unhappy    not happy   not unhappy   happy
```
xxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxx   not happy
xxxxxxxx   unhappy

- Law of excluded middle does not make sense in this picture.
- **Not unhappy** counts as an additional alternative
- 4 alternatives are matched with the measurement scale
- $\langle$ **happy**$_A$, **not unhappy**$_I$, **unhappy**$_E$, **not happy**$_O\rangle$

Negative Strengthening (NegS)
medium size elimination not possible

(12)   He is not brilliant (O) *$\hookrightarrow$ He is idiotic (E)

-----------/-/---------------/-/-------------/-/---------$\rightarrow$
idiotic   not brilliant   intelligent   brilliant
xxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxx   not brilliant result of strengthing
xxxxxxxx   idiotic

- Law of excluded middle does not make sense anymore in this picture.
- **intelligent** counts as an additional alternative
- 4 alternatives are matched with the measurement scale
- $\langle$**brilliant**$_A$, **intelligent**$_I$, **idiotic**$_E$, **not brilliant**$_O\rangle$

Conclusion

- The difference between sentential negation and affixal negation is a scope difference.
- There is only one negation: truth value reversal.
- Either a comparison relation is denied (introduced by the adjective) or the POS operator.
- It could be that **not** were type flexible all the way through: even a degree modifier. **un-** is never sentential (scopally inert) for type reasons. I proposed a POS operator with existential force and **not** is no degree modifier, by applying the Square of Opposition. The NegHypothesis remains valid.
- The account might be applied to the realm of scalar impicatures and makes the correct predictions.
- The availability of the implicatures and negative strengthening is a contextual effect of the restriction of the positive operator (the distance between two antonyms) and this matches the findings in Gotzner et al. (2018). Distance matters!

<div align="center">

Thank you!

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