

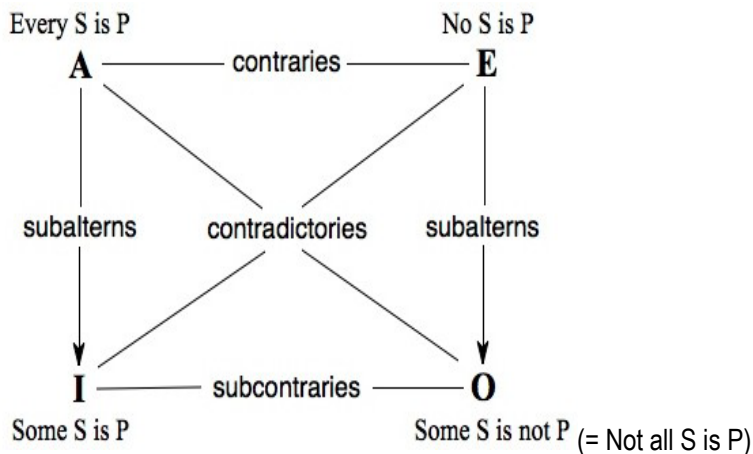
Negation and scalar implicature: the first 750 years

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O me no O's

—Ben Jonson (1609), “The case is altered”, V.i

1. Lexicalization and the three-cornered Square



[See Parsons 2017, <https://plato.stanford.edu/entries/square/>; Horn 2018a, <https://plato.stanford.edu/entries/contradiction/>]

The missing O corner and geometric asymmetry: first sighting

St. Thomas Aquinas, c. 1273 (give or take):



For the universal negative **E** “the word ‘no’ [*nullus*] has been devised [!] to signify that the predicate is removed from the universal subject according to the whole of what is contained under it”, but when it comes to the particular negative **O**, we find

there is no designated word, but ‘not all’ [*non omnis*] can be used. Just as ‘no’ removes universally, for it signifies the same thing as if we were to say ‘not any’ [= ‘not some’], so also ‘not all’ removes particularly inasmuch as it excludes universal affirmation.

(Thomas Aquinas, *in Arist. de Int.*, Lesson X, in Oesterle 1962: 82-3)

From the sublime to the...chatbot [ChatGPT3, accessed January 2023]:

Q: *Why isn't there a single word meaning "not all"?*

A: There is no single word in English that means "not all." However, you can use the word "some" to indicate that not all of a group or category is being referred to. For example, you could say "Some of the students are absent" to indicate that not all of the students are present. Similarly, you could use the word "none" to indicate that none of a group or category is being referred to. For example, you could say "None of the cookies are chocolate" to indicate that not any of the cookies are chocolate.

??? So *not all* = *some*? *not all* = *none*? So *some* = *none*? Second try:

A: There is no single word in English that means "not all", but you can use the phrase "not all" to convey this meaning. For example, you could say "Not all dogs are friendly" to mean that not every single dog is friendly. Alternatively, you could use the word "some" to convey the same meaning, as in "Some dogs are friendly".

Third try:

A: There is actually a word that means "not all," and that word is "not every." For example, you could say "Not every person in the room liked the movie," which means "Not all of the people in the room liked the movie." The word "not" functions as a negation, while "every" means "all," so "not every" means "not all." Is there something else you would like to know about this?

☛ Languages tend not to lexicalize operators made redundant by scalar implicature, whence the absence of **nall*, **nand*, and other values mapping to the **O** corner. (= "L'histoire d'***O**") (Horn 1972; cf. Horn 1989, 2012 but also Hoeksema 1999, Béziau 2003, Jaspers 2005, Moeschler 2007, Seuren & Jaspers 2014, and others for diverse explanations)

{**A, E, I**} >> {**A, E, I, O**} but also (on the same economy grounds) {**A, E, O**} >> {**A, E, I, O**}
But given the markedness of negative expressions, we obtain {**A, E, I**} >> {**A, E, O**}
—and indeed, typological studies of inventories show **A, I** >> **E** >> **O**

- Asymmetries in **lexicalizability** (horizontal and vertical)
- Asymmetries in **expressibility** (beyond lexicalization)
- Asymmetries in **frequency**/"markedness"
- Asymmetries in **learnability**

Relation between these asymmetries and **semantic universals**

Semantic universals hold because expressions satisfying the universal are easier to learn than those that do not... [M]eanings that are easier to learn are more likely to be lexicalized. (Steinert-Threlkeld & Szymanik 2019: 4)

☛ An ongoing explosion of work on the quaternary asymmetries:

Katzir & Singh 2013	Uegaki 2021
Steinert-Threlkeld 2019	Enguehard & Spector 2021
Steinert-Threlkeld & Szymanik 2019	Züfle & Katzir 2021
Carcassi, Steinert-Threlkeld & Szymanik 2021	Carcassi & Sbardolini 2022

☛ The role of polarity (or valence)

A, I >> **E** >> **O**: Many languages lexicalize **A** and **I** but neither of the negative values

Preference for **I** over **O**: if one of the two subcontraries lexicalizes, it will always be **I** (*some, or*), never **O** (*not all, not and*).

Lexicalizations of **E** (as opposed to **A**) tend to be morphologically and lexically complex, although there are many languages with **E** lexicalizations (*no, none, nothing, never, nor*; Fr. *aucun, ni, rien, jamais, ni*), and some of these are morphologically simple

Geurts (2003: 246), citing Horn 1989 and other work on the markedness of negation:

“Psychologically, negative expressions take longer to process, cause more errors, and are harder to retain than positive ones.”

- We can extend this to other mon↓ operators (*at most n* harder to process than *at least n*)

Katsos et al. (2016) on typological patterns in acquisition of quantifiers: **A >> I/E >> O**

TOTALITY (**A >> I**; **E >> O**)

in 26/31 languages, children did better with *all* than *some*

in 29/31 languages, children did better with *none* than *some not*

POLARITY (**A >> E**; **I >> O**)

in 30/31 languages, children did better with *all* than *none*

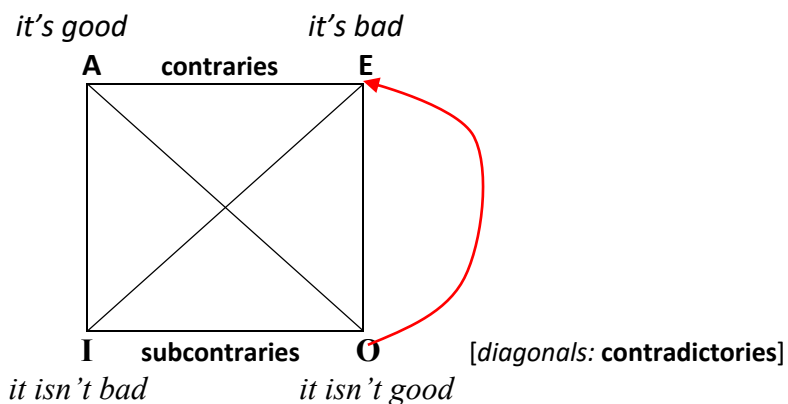
in 28/31 languages, children did better with *some* than *some not*

2. Negative strengthening and the vertical asymmetry of the Square

MAXCONTRARY: The tendency for contradictory (exhaustive) negation to be interpretatively strengthened to a contrary (antonymous) meaning

Contrariety tends to be maximized in natural language.
Subcontrariety tends to be minimized in natural language.

- **O > E** drift: expressions that “ought” to have **O** semantics (contradictory to **A**) often end up denoting a contrary, **E**-type value, reflecting **MAXCONTRARY**



SINGULAR SQUARE OF OPPOSITION

(Horn 2014, 2017, based on Aristotle's *Prior Analytics I*, Chapter 46)

- Formal negation tends to be understood as interpretatively strengthened from (mere) **contradictory** opposition to **contrariety**, given the appropriate pragmatics.
- The speaker tends to weaken the force of **contrary** negation by packaging it as **contradictory** negation, motivated in part by concerns of politeness (respecting “negative face” as with euphemisms; cf. Goffman 1956, Brown & Levinson 1987, Horn 1989, Ruytenbeek et al. 2017, Gotzner & Mazzarella 2020, among others).

The essence of negation is to invest the contrary with the character of the contradictory.....From ‘he is not good’ we may be able to infer something more than that ‘it is not true that he is good’...Compare our common phrase ‘I don’t think that’—which is really equivalent to ‘I think that ___ not’. (Bosanquet 1888: 306ff.)

Litotes (affirmation of the contrary; see Horn 2017 for overview and references)

I don't like pineapple pizza is strengthened to \approx *I dislike pineapple pizza*

They aren't happy is strengthened to \approx *They're unhappy/sad*

Neg-raising (see Horn 2020 for overview and references)

I don't want to see you anymore conveys *I want to not see you anymore*

I don't think I'll survive SALT conveys *I think I won't survive SALT*

3. Communicative efficiency

- General preference for efficient communication reflected in semantic change and in lexical domains of color and kinship terms and folk taxonomies (Kemp et al. 2018) and specifically in the acquisition of quantifiers (Katsos et al. 2016).

- The logical lexicon—especially within the domains of determiners, generalized quantifiers, and propositional connectives but also quantitative adverbs and to a considerable extent the modalities—is shaped by a trade-off or tug-of-war between informativity and cost:

Languages maximize communicative efficiency through a trade-off between **informativity** and some notion of **lexical complexity**.

(Enguehard & Spector 2021: 16-17)

Quantifiers in natural language optimize the **simplicity/informativeness** trade-off.

(Steinert-Threlkeld 2019, paper title)

- Interaction of informativity (including implicature) vs. cost: the “H/KS/U” approach (Züfle & Katzir 2021), but continuing debate on whether and how to allow for extra cost for negation

- Enguehard & Spector (2021): **I, O** equally informative pragmatically but **I** has greater SURPRISAL VALUE; speaker prefers message whose literal meaning is more informative/less costly

Informativity/Clarity vs. **Ease/Laziness/Economy/Comfort**: a brief history of the tug-of-war

- cf. Curtius 1870, Paul 1889, von der Gabelentz 1891, Zipf 1949, and many others on the tug-of-war (Jespersen 1941: 391) between the **informativeness/distinctness/clarity of the message** (Zipf's **hearer's economy**) and the **drive to reduce effort** (Zipf's **speaker's economy**).



John Morgan (1822–1855), “The Tug-of-War”

Brevis esse laboro, obscurus fio. ‘I strive to be brief, I become obscure’
—Horace, *Ars Poetica* 25 (19 BCE)

Maximize meaning, minimize means.
—maxim of graphic designer Abram Games (1914-1996)

The linguist must keep in mind two ever-present and antinomic factors: first, the requirements of communication, the **need for the speaker to convey his message**, and second, **the principle of least effort, which makes him restrict his output of energy, both mental and physical**, to the minimum compatible with achieving his ends. (Martinet 1962: 139)

The speaker always tries to optimally **minimize the surface complexity** of his utterances while **maximizing the amount of information** he effectively communicates to the listener. (Carroll and Tanenhaus 1975: 51)

The task of category systems is to **provide maximum information** with the **least cognitive effort**. (Rosch 1978: 28)

Horn (1984, 1989): Zipfo-Gricean dualistic model, a trade-off between the hearer-oriented **Q Principle** (essentially “**say enough**”), collecting Grice’s Quantity-1 along with Avoid ambiguity/vagueness, and the speaker-oriented **R Principle** (“**don’t say too much**”), collecting Quantity-2, “Be relevant”, and “Be brief”

Maxim of Quantity (Grice [1967]1989: 26):

QUANTITY-1. Make your contribution as informative as is required...

QUANTITY-2. Do not make your contribution more informative than is required.

Kiparsky (2005: 114): OT framework invoking two defeasible constraints:

ECONOMY: Avoid complexity

EXPRESSIVENESS: Express meaning

Among equally expressive expressions, the simplest is optimal.

Among equally simple expressions, the most expressive is optimal.

- Keeping informativity constant, languages tend to **minimize CONCEPTUAL COMPLEXITY while also minimizing USAGE COMPLEXITY**, resulting in a trade-off between those two forces
—Carcassi & Sbardolini 2022, recalling Martinet on restricting mental and physical energy.

What one may call the economy of language is this permanent search for equilibrium between the contradictory needs which it must satisfy: communicative needs on the one hand and articulatory and mental inertia on the other, the two latter in permanent conflict. (Martinet 1960: 169)

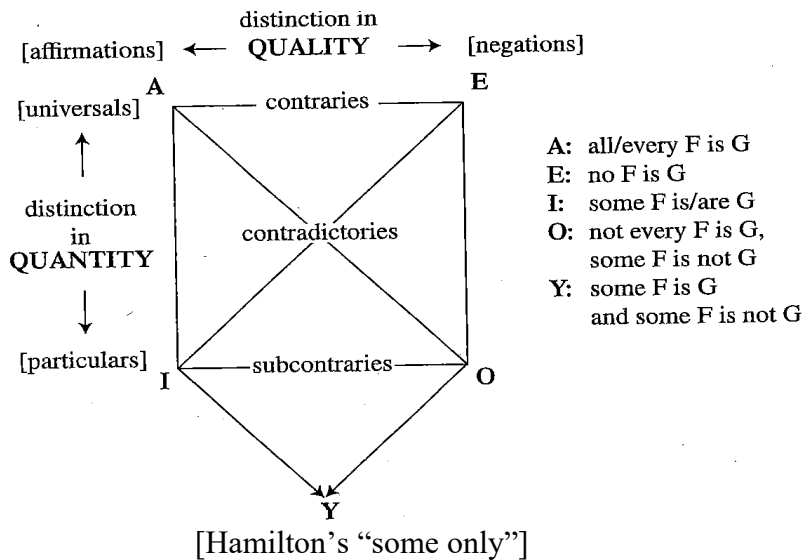
3. The diverse geometries of opposition

Sir William Hamilton of Edinburgh (1860: 254ff.):

There are two distinct senses of some, the INDEFINITE (*at least some [I]*) and the SEMI-DEFINITE (*some but not all [Y]*), with the latter as the default:

Some, if not otherwise qualified, means some only—this by presumption... In reality and in thought, every quantity is necessarily either all, or none, or some. Of these the third...is formally exclusive of the other two.

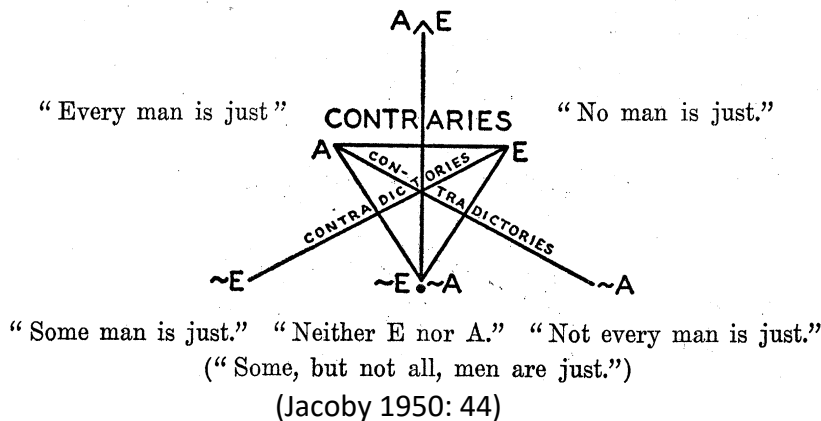
Extending the square: The Pentagon of Opposition



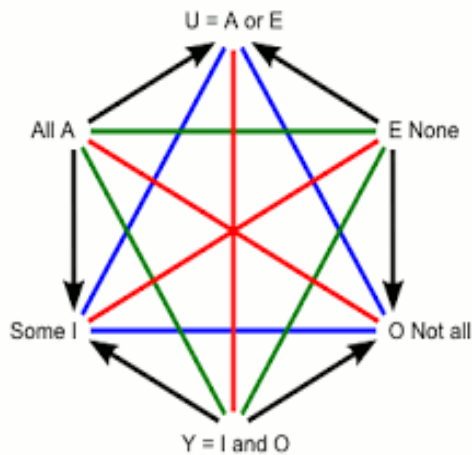
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From Pentagon to Hexagon (Cf. Jacoby 1950; Sesmat 1951; Blanché 1952, 1969)

"Either A or E."



The definitive hexagon: the Blanché Star

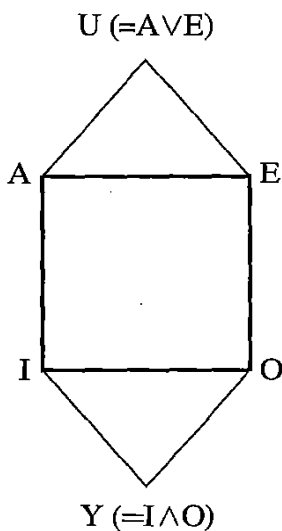


(Blanché 1969: 56; cf. Blanché 1952, 1953)

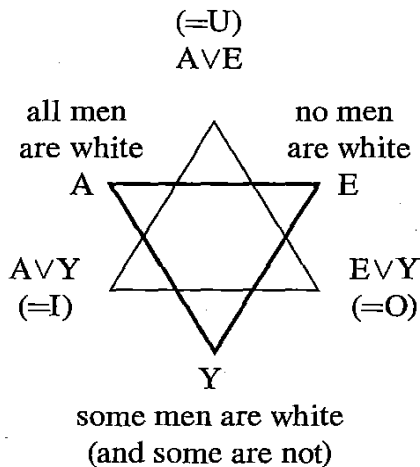
For much more on hexagons and other geometries of opposition, including $n > 2$ -dimensional variants, see Smessaert & Demey 2014 and papers in Béziau & Payette 2012.

August De Morgan (1858: 121) on the epistemic constraint on opposition:

There are three ways in which one extent may be related to another...: complete inclusion [A], partial inclusion with partial exclusion [Y], and complete exclusion [E]. This trichotomy would have ruled the forms of logic, if human knowledge had been more definite...As it is, we know well the grounds on which predication is not a trichotomy, but two separate dichotomies.



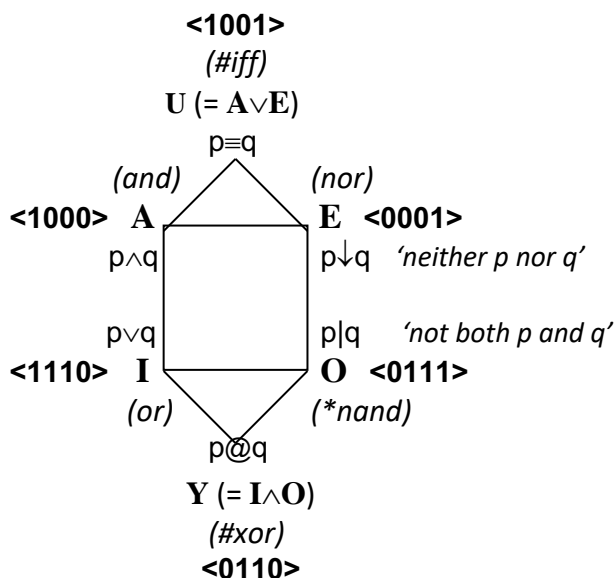
Logical hexagon
(Blanché et al.)



Logical Magen David
(Horn 1990)

Hexagon of binary connectives

based on Horn 2012: 406; see now Uegaki 2021, Carcassi & Sbardolini 2022:
Minimize complexity while maximizing informativity **given scalar implicature**



The missing O: some relevant paradigms

	DETERMINERS/ QUANTIFIERS	QUANT. ADVERBS	BINARY QUANTIFIERS	CORRELATIVE CONJUNCTIONS	BINARY CONNECTIVES
A:	all α , everyone	always	both (of them)	both...and	and
I:	some α , someone	sometimes	one (of them)	either...or	or
E:	no α , no one (=all-/-some)	never (=always-)	neither (of them) (=both-/-either)	neither...nor (=[both...and]-)	nor (=and-)
*O:	*nall α , *neverone (=some-/-all)	*nalways (=-always)	*noth (of them) (=either-/-both)	*noth...nand (=[either...or]-)	*nand (=and-/-or)

N.B.: These observations apply to natural languages only, both spoken and signed; **NAND GATES** in electronic circuitry cover precisely the forbidden **O**-vertex meaning, just as it has often proved useful to define a set-theoretic operator **XOR** based on exclusive disjunction:

“Exceptions” to the constraint on lexicalizing **O**, **Y**, or **U** connectives

- **NAND**, a Boolean operator in programming languages:
 $p \text{ NAND } q$ is true iff p and q are not both true **<0111>**
- $x \in A \text{ XOR } B$ iff $x \in A \cup B$ & $x \notin A \cap B$ **<0110>**
(see Gazdar & Pullum 1976, Horn 1989 on why *aut* and other putative instances of exclusive disjunction are not in fact **<0110>** truth functions)
- $A \text{ IFF } B$ or $A \equiv B$ **<1001>** lexicalizes in propositional calculus but not in ordinary language

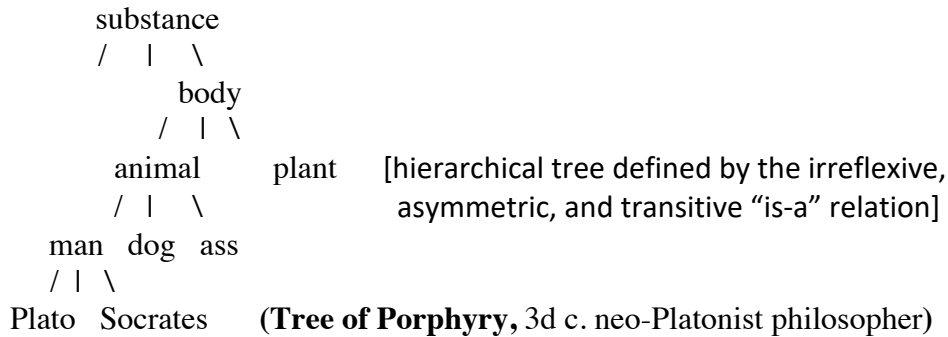
Gazdar & Pullum (1976): Noncommutative connectives cannot be truth-functional connectives.

But noncommutative *if...then* is simple and learnable, although arguably not a truth function **<1011>**, while *iff*, which is a commutative truth function **<1001>**, is never simple.

- Another factor in predicting which values are capable of lexicalizing: **MONOTONICITY**

5. Monotonicity (the first 750 years (+/-))

The roots of monotonicity in the medieval theory of *suppositio* (cf. *Dictum de omnis et nullo*)



Upward and downward inference (as in Billingham’s *Speculum Puerorum* and Alnwick’s *De Veritate et Falsitate Propositionis* in de Rijk 1982; see the discussion of VERTICAL RULES in Sánchez Valencia 1994) are governed by two key *regulae*:

- ☞ There is a valid consequence from an inferior to its superior (*ab inferiori ad suum superius*) in the absence of negation or a term with the force of negation. [mon↑]
- ☞ There is a valid consequence from a superior to its inferior (*a superiori ad suum inferius*) with a **preposed negation** or distributive term. [mon↓]

“**PREPOSED NEGATION**”—because it’s only material that falls within the scope of the relevant operator that creates the flip in the direction of inference (= scale reversal):

Non animal currit; ergo non homo currit vs.
Homo non currit; ergo animal non currit.

“In general, downward monotonicity is seen as a generalization of properties of negation” (Sánchez Valencia 1994: 172), as (independently) for Fauconnier and Ladusaw

Terms with the **FORCE OF NEGATION** (*termini habentes vim negationis*) include:

- overt negation
- the comparison clause of a comparative
- restrictors of universals [*Omnis homo currit, ergo omnis homo albus currit*]
- exclusives (*tantum* ‘only’) and exceptives (*præter* ‘except, but’)

Similar inferential mechanisms are found in Ockham and date back to anonymous authors of the 12th and early 13th centuries

Key idea: Expressions with the force of negation block upward inferences and allow downward ones within their scope

Note that the dual verticals (**A/I**; **E/O**) constitute monotonically related scalar values, while the values corresponding to the **Y** and **U** vertices are non-monotonic and non-scalar.

Steinert-Threlkeld & Szymanik (2019: 4) and others present Barwise & Cooper (1981)’s **MONOTONICITY UNIVERSAL** as the condition **All simple determiners are monotone.**

Carcassi et al. (2021) argue that the monotonicity universal arises via learning biases as demonstrated by neural network models, constituting a substantive semantic universal.

But Barwise & Cooper’s monotone universal (1981: 187) is actually given more weakly as a disjunctive constraint on NPs (generalized quantifiers):

The simple NPs of any natural language express monotone quantifiers or conjunctions of monotone quantifiers. (emphasis added)

(i) The strong version of the universal would correctly rule out **Y** values (*some but not all CN*) and **U** values (*some or all CN*) along with e.g. *an even number of women, all but one man*

(ii) Apparent issue with *a few CN* as a counterexample to the strong version can be dissolved

- *a few* forms upward monotone NPs: see arguments in Horn 2018b based on distributional evidence and citing earlier arguments in Ducrot 1970 and Moxey & Sanford 1993, as well as remarks by Barwise & Cooper (citing Horn 1972) themselves:

It is likely that the $\text{mon}\uparrow$ reading is the only one that should be accounted for by the semantics, conversational implicature explaining **the illusion of a non-monotone reading**.

e.g. B&C's conjunction test: $[\text{mon}\uparrow \text{ and } \text{mon}\uparrow]$, $[\text{mon}\downarrow \text{ and } \text{mon}\downarrow]$ vs. $[\text{mon}\uparrow \text{ but } \text{mon}\downarrow]$
many women and/#but most men; few women and/#but no men
many women but/#and few men; many women and/#but a few men

- parallel diagnostics show that *only a few* is non-monotone (while obviously non-simple)

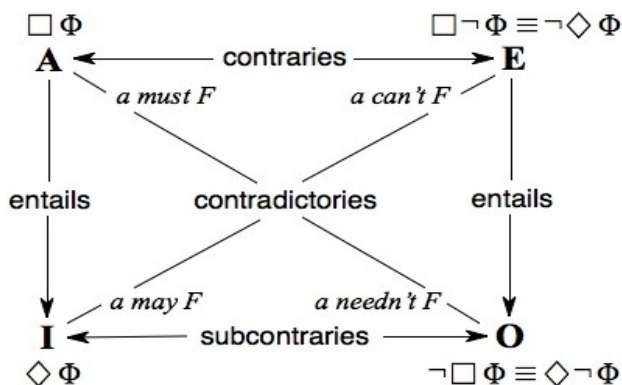
(iii) Remaining issue of cardinal/numerical NPs: *at least n* vs. *exactly n* readings (*3 little pigs, 12 days of Christmas*): see Geurts & van der Silk 2005 on the relevance of cardinals for the monotonicity universal and Koenig 1991, Horn 1992, Geurts 1998, Bultinck 2005, Hurewitz et al. 2006, Breheny 2008, and Solt & Waldon 2018 on contrasts between cardinals vs. “inexact” scalars in distribution, processing, and acquisition

Non-monotone disjunctive **U** values (e.g. *all or none*), while easy to process, are cognitively complex and lexically unattested (Jaspers 2005; Carcassi et al. 2021).

Non-monotone **Y** values—e.g. “two-sided *some*” (*some but not all*); exclusive *or*—are often conveyed via scalar implicature but (pace Sir William Hamilton) not literally represented.

Similar asymmetries obtain elsewhere among logical operators that can be superimposed on the square—contraries lexicalize but negative subcontraries don't—but the strength of the constraint varies depending on the categories and the factor of closed- vs. open-class inventories

Modal square (Horn 1989, van der Auwera 1996)



Monotonicity and scales

<some, many, most, all>	<not all, few/not many, no(ne)>
<sometimes, often, usually, always>	<not always, rarely/seldom, never>
<possible, likely, certain>	<not certain, unlikely, impossible>
<may/can, should, must>	<needn't, shouldn't, can't/mustn't>

Lexicalization asymmetry: powerful but not absolute

- (1) A priest can not marry. [both Catholic **E** ($\neg\Diamond$) and Anglican **O** ($\Diamond\neg$) readings]
(2) A priest {can't/cannot marry}. [only Catholic **E** ($\neg\Diamond$) reading], no **O** ($\Diamond\neg$) reading]
(3) A priest needn't marry. [**O** ($\neg\Box$) reading]

... but note the register and flavor restriction and the NPI character of *need*

Similarly, compare *impossible/impossibility* vs. *unnecessary/*unnecessity*

- In some languages, including Turkish, ASL, and Langue signée française, we find an opaque **E**-valued modal negation synchronically distinct from both possibility and necessity; its **O** counterpart (\approx *needn't*) is transparent and non-lexicalized (= 'not' + 'must/have to')

Expressibility: Variable-force modals allowing **A/I** interpretations invariably express **E** rather than **O** meanings under negation, instantiating MaxContrary

- (4) a. OE *motan* (Goossens 1987: 33): could denote permission, ability, or obligation, but:

Hit is halig restendæg; ne most ðu styrgan þine beddinge.
it is holy rest-day not may/can/must thou move thy bed
'This is a holy day; you(sg.) {may not/can't/mustn't(**E**)/*can not (**O**)} move your bed'
•see Yanovich 2006 for elaboration

- b. Nez Perce *o'qa* (Deal 2011: 573) can denote possibility/necessity in mon \uparrow contexts, but:

The semantic intuitions of speakers concerning negated *o'qa* sentences are unambiguous. Such sentences have only $\neg\Diamond$ (not-possible) readings... Speakers firmly reject $\Diamond\neg$ (possible-not) and logically equivalent $\neg\Box$ (not-necessary) translations of negated *o'qa* sentences, and reject *o'qa*-sentences when presented with $\Diamond\neg$ or $\neg\Box$ as elicitation prompts.

(Similar facts hold in Kinande, Ecuadorian Siona, Swedish: Variable force modals lose their variability under negation and express only **E** meanings—Lydia Newkirk, p.c.)

- In many languages, negated causatives express (as their primary or only meaning) prohibition (CAUSE-NOT rather than NOT-CAUSE)

Cf. Horn 1989, 2012, 2014 for more on lexicalization asymmetries and **O** > **E** drift

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