Arguments, Conditionals, and Context

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1 Examples of arguments

1.1 ... in political discourse

(1) Our cruel and unrelenting enemy leaves us only the choice of brave resistance or abject submission. We have, **therefore**, to resolve to conquer or die. (George Washington 1776, General Orders, 2 Jul.)

(2) All free men, wherever they may live, are citizens of Berlin, and **therefore**, as a free man, I take pride in the words,'Ich bin ein Berliner. (J.F. Kennedy, 1963 Speech at West Berlin City Hall, Rudolf Wilde Platz.)

(3) I cannot deliver the mandate on which I was elected by the Conservative Party. **Therefore**, I am resigning as leader of the Conservative Party. (Truss in her short resignation speech.)

1.2 ... in literary discourse

(4) The impossible could not have happened. **Therefore**, the impossible must be possible despite the appearances. (Agatha Christie, Murder on the Orient Express).

(5) She is a woman, **therefore** may be wooed She is a woman, **therefore** may be won; She is Lavinia, **therefore** must be loved. (Shakespeare, Titus Andronicus, Act 2 Scene 1).

(6) Crime is common. Logic is rare. **Therefore**, it is upon the logic rather than upon the crime that you should dwell. (A.C. Doyle 1892 The Adventures of Sherlock Holmes).

1.3 ...in philosophical discourse

(7) I think, **therefore** I am. (Descartes, Meditations).

(8) When I see a billiard ball moving towards another, my mind is Immediately carried by habit to the usual effect, and anticipates my sight conceiving the second ball in motion. There is nothing in these objects, abstractly considered, and independent of experience, which leads me to form any such conclusion; . . . It is not, **therefore**, reason which is the guide of life, but custom. (David Hume, Treatise of Human Nature).

(9) Your objections, I must freely tell you, are no better than the abstruse cavils of those philosophers who denied motion; and ought to be refuted in the same manner, by illustrations, examples, and instances, rather than by serious argument and philosophy. **Suppose, therefore, that an articulate voice were heard in the clouds**, much louder and more melodious than any which human art could ever reach . . . Could you possibly hesitate a moment concerning the cause of this voice? And must you not instantly ascribe it to some design or purpose? (David Hume, Dialogues concerning Natural Religion). [→ Notice the argumentative conclusion].

1.4 ...in scientific discourse

(10) **Suppose** we have a set of 'elements' of some sort. **Suppose** that these elements possess one or more fundamental identifying characteristics, analogous to the coordinates of a point... **Suppose** we find that no two elements of the set possess identically the same set of defining values. **Suppose** finally that the elements of the set are such that, no matter what numerical values we define by these an actual element of the set, that corresponds to this particular collection of values. Our elements **then** share with the real number system the property of sharing no holes, of constituting a continuous possession in every dimension which we possess. We **then** have a continuum. (J.M. Bird, Einstein's theory of relativity and gravitation, 1921 p. 148-149).

1.5 ...in religious discourse

(11) But seek first the kingdom of God and His righteousness, and all these things will be added unto you. **Therefore**, do not worry about tomorrow, for tomorrow will worry about itself. Today has enough trouble of its own. (Matthew 3:64).

(12) Whoever is faithful with very little will also be faithful with much, and whoever is dishonest with very little will also be dishonest with much. **Therefore**, if you have not been faithful with worldly wealth, who will entrust you with true riches? (Luke 3:64).

2 A taxonomy of arguments

Categorical arguments

(13) a. It is raining. **Therefore**, streets are wet. DECLARATIVE CONCLUSION
b. It is raining. Will the street be, **therefore**, wet? INTERROGATIVE CONCLUSION
c. It is raining. **Therefore**, take the umbrella! IMPERATIVAL CONCLUSION

Suppositional arguments

(14) a. Suppose it is raining. **Then**, streets are wet. DECLARATIVE CONCLUSION
b. Suppose it is raining. **Then**, will the street be wet? INTERROGATIVE CONCLUSION
c. Suppose it is raining. **Then**, take the umbrella! IMPERATIVAL CONCLUSION

Complex arguments

(15) **Suppose** it is raining. **Then** the streets are wet. **Therefore**, if it is raining, the streets are wet. (CONDITIONAL PROOF)

(16) Whoever committed the crime left by the window. Anyone who had left by the window would have mud on his shoes. **Suppose** the butler committed the crime. **Then** he left by the window. In that case, he has mud on his shoes. **So**, if the butler committed the crime, he has mud on his shoes. (CONDITIONAL PROOF)

(17) It is raining. **Therefore**, suppose you forget the umbrella. You will then get wet. ARGUMENTATIVE CONCLUSION

(18) Maria is either from Turin or from Madrid. **Suppose** she is from Turin. **Then** she is Italian. **Suppose** instead she is from Madrid. Then she is Spanish. **Therefore**, she is either Italian or Spanish. (ARGUMENT BY CASES)

(19) Creepy Calabresi got off the plane in either Chicago, Kansas City, or Las Vegas. **Suppose** he got off in Chicago. **Then** we would have called his brother. But his brother wants to get rid of Creepy and he would have tipped off the feds. **Suppose** Creepy got off in Kansas City. **Then** he would have called his girl-friend. But his girl-friend is working for the IRS now, and she would have tipped off the feds.**Suppose** Creepy got off the plane at Las Vegas. **Then** he would have called the Fettucini Kid. But the Fettucini Kid has been arrested and the fuzz would have a stoolie taking the phone calls, and he would have tipped off the feds. **So** someone has tipped off the feds. (ARGUMENT BY CASES)

(20) **Suppose** there is a largest prime number p. **Then** p! + 1 is larger than p. But p! + 1 is prime, contradiction. **Therefore**, there is no largest prime number. (REDUCTIO)

3 Affinities with conditionals

3.1 Used often interchangeably

(21) a. **Suppose** that the butler did it. **Then** the gardener is innocent.
b. If the butler did it, then the gardener is innocent.

(22) a. **Suppose** Oswald didn't kill Kennedy. **Then** someone else did. (INDICATIVE MOOD)
b. If Oswald did not kill Kennedy, someone else did. (INDICATIVE MOOD)

(23) a. **Suppose** Oswald hadn't killed Kennedy. **Then**, someone else would have. (SUBJUNCTIVE MOOD)
b. If Oswald hadn't killed Kennedy, someone else would have. (SUBJUNCTIVE MOOD)

3.2 Similar patterns of modal subordination

Persistence

(24) a. If a wolf comes in, we will use a gun. If we manage to shoot, we will be safe. If we bury the body, nobody will find out.
b. Suppose a wolf comes in. We will use a gun. Suppose we manage to shoot. We will be safe. Suppose we bury the body. Nobody will find out.

Reversibility

(25) a. If it is raining, the park will be wet. If it is not, then the park will be dry.
b. Suppose it's raining. The park will be wet. Suppose it isn't. The park will be dry.

4 First Observation: Coordination

Background: in a game of dice, only and all even numbers win.

(26) a. ✓ If the die comes up 2 and if the die comes up 4, Ben will win. (Starr 2014, Khoo 2022).
b. # Suppose the die comes up 2 and suppose the die comes up 4. Then, Ben will win.

5 Second Observation: Behavior after categorical statements and in *modus ponens* arguments

(27) a. Mark might be Italian. ✓ If he is Italian, then he must be European. Therefore, Mark is European.
b. Mark might be Italian. ✓ Suppose he is Italian. Then he must be European. Therefore, Mark is European.

(28) a. Mark is Italian. ✓ If he is Italian, then he must be European. Therefore, Mark is European.
b. Mark is Italian. ?? Suppose he is Italian. Then he is European. Therefore, Mark is European.

(29) a. The car crashed at greater than 35mph. ✓ If the car crashed at greater than 35mph, then the airbag must have gone off. Therefore, the airbag must have gone off.
b. The car crashed at greater than 35mph. ?? Suppose the car crashed at greater than 35mph. Then the airbag must have gone off. Therefore, the airbag must have gone off.

6 Third Observation: Conditional proof

(30) ✓ Suppose John attends the party. Then Georgi will attend. Therefore, if John attends, Georgi will attend. (CONDITIONAL PROOF, SUMMARY USES)

(31) ??Suppose John attends the party. Then Georgi will attend. Therefore, suppose John attends the party. Then Georgi will attend. (DOUBLE SUPPOSING ARGUMENT)

(32) If John attends the party, Georgi will attend. Therefore, if John attends the party, Georgi will attend. (TRIVIAL ARGUMENT)

(33) ✓ Suppose the butler committed the crime. Then he left by the window. In that case, he has mud on his shoes. So, if the butler committed the crime, he has mud on his shoes. (CONDITIONAL PROOF, SUMMARY USES)

(34) # Suppose the butler committed the crime. Then he left by the window. Then, he has mud on his shoes. So, suppose the butler committed the crime. Then, he has mud on his shoes. (DOUBLE SUPPOSING ARGUMENT)

SIMPLE CONDITIONAL PROOF

Suppose P. Then, Q. Therefore, if P then Q.

THE TRIVIAL ARGUMENT

If P, then Q. Therefore, if P then Q.

THE DOUBLE SUPPOSING ARGUMENT

Suppose P. Then Q. Therefore, suppose P. Then Q.

7 Supposing, supposing

(35) a. ✓ Supposing that Basic Law V is true, Frege got a contradiction.
b. ✓ Supposing that Basic Law V is true, one gets a contradiction.
c. ✓ Supposing that Basic Law V is true, a contradiction follows.

(36) a. ✓ On the supposition that Basic Law V is true, Frege got a contradiction.

b. ✓ On the supposition that Basic Law V is true, one gets a contradiction.
c. ✓ On the supposition that Basic Law V is true, a contradiction follows.

(37) a. ## If Basic Law V is true, Frege got a contradiction.
b. # If Basic Law V is true, one gets a contradiction.
c. ??If Basic Law V is true, a contradiction follows.

(38) a. ✓ If supposing that Basic Law V is true, you got a contradiction, you ought to conclude that Basic Law V is not true.
b. ??If you got a contradiction if Basic Law V is true, you ought to conclude that Basic Law V is not true.

8 Summarizing

Observation #1 Conjunctions of incompatible *ifs* are fine, whereas conjunctions of incompatible *suppose* are not.

Observation #2 'Suppose' is less tolerated than if-clauses after the categorical statement of their content.

Observation #3 If-clauses are much preferred to 'suppose'-clauses as conclusions of arguments by conditional proofs.

Observation #4 Categorical arguments differ from conditional discourse in that their conclusions are discharged of the premises. But the mechanism of discharging of the premises in natural language discourse is not accomplished through the same mechanisms that natural deductive systems employ.

Observation #5 Conditionals and argument share several affinities (I-III).

9 Anaphoricity of argument connectives

1. They need an antecedent:

   (39) a. ??Therefore/Hence/Thus, we should leave.
   b. ??Therefore/Hence/Thus, streets are wet.
   c. ??Therefore/Hence/Thus, either it is raining or it is not raining.

2. Like other anaphors, the linguistic antecedent for 'therefore' does not need to be the most immediate one:

   (40) Suppose Mark went to the grocery store this morning. (Have you been? They have all sorts of exotic fruit.) Then he bought some dragon fruit. Therefore, he can make an exotic fruit salad.

3. Like anaphors, it can be ambiguous what the antecedent is:

   (41) Either it's raining or it's not. Suppose it's raining. Then you should take the umbrella. Suppose it is not raining. Then taking the umbrella will do no harm. <u>Therefore</u>, you should take an umbrella.
   a. CATEGORICAL: you should take an umbrella regardless of whether it's raining or not.
   b. SUPPOSITIONAL: you should take an umbrella also assuming it's not raining.

4. Donkey sentences involving 'therefore':

   (42) a. Whenever one believes a certain view, one has to believe that its consequences are therefore true.
   b. If one derives a contradiction from a claim, one may infer that it is therefore false.

10 Towards A Semantics for Argumentative Discourse

10.1 Syntax: Discourses and Labeled Sentences

Let $\therefore$ stands for argument connectives such as 'therefore' and 'then' and let $+$ stands for a suppositional operator. Our syntax for UNLABELED SENTENCES is as follows:

$$\phi ::= p \mid \neg\phi \mid (\phi \wedge \phi) \mid (\phi \vee \phi) \mid (\phi \rightarrow \psi) \mid \Box\phi \mid \Diamond\phi$$
$$\sigma ::= \phi \mid \therefore\phi \mid +\phi \mid \therefore + \phi.$$

A LABELED SENTENCE is a pair of the form $\langle n, \phi\rangle$, which we write as $n\colon \phi$ for short.

- We write $\langle n_1, \ldots, n_k\rangle$ in decimal form as $n_1.n_2.\ldots.n_k$
- We use "0" to stand for the empty tuple $\langle\rangle$ (the "categorical" label)

A DISCOURSE is a sequence of labeled sentences. Understanding discourses in terms of labeled sentences enables to track anaphoric relations that 'therefore' establishes in discourse.

11 Remarks

We can capture the anaphoric behavior of 'therefore' by imposing some additional constraints on the current syntax.

- One constraint is that the first labeled sentence of a discourse cannot be of the form $0\colon \therefore\phi$. So, (43) is not a proper discourse:

  (43) ??Therefore, we should leave.

  Furthermore, a labeled sentence of the form $n\colon \therefore\phi$ (where $n \neq 0$) cannot occur unless there is a supposition prior to it that introduces the label n into the discourse. (This is parallel to the constraint that an anaphoric expression cannot have an index that does not appear before.)

- Another constraint on the structure of discourses might be that suppositions cannot be "idle" — i.e., introduced without a consequent (or without a discourse whose first element contains its label as an initial segment). This rules out, e.g., discourses of the form $n\colon +\phi, n\colon +\psi$, where the supposition ϕ is introduced but not used. Thus, a sequence of suppositions must be interpreted as introducing additional levels. To illustrate, sequences of suppositions like (44) sound marked since the second supposition is interpreted in the scope of the first (as in (44a)) rather than as a separate supposition (as in (44b)).

  (44) Suppose physicalism is true. ??Suppose physicalism is false...
  a. Suppose$_1$ physicalism is true. Suppose$_{1.1}$ physicalism is false...
  b. Suppose$_1$ physicalism is true. Suppose$_2$ physicalism is false...

  Because the second supposition is interpreted within the scope of the first, as in (44a), and cannot be interpreted as in (44b), we have explained why (44) is infelicitous.

- A final constraint is a prohibition on "crisscrossing" labels (cf. the "right frontier constraint" in SDRT). Essentially, this means that within a suppositional environment, we cannot refer back to other suppositions on the same level. This rules out, e.g., discourses of the form: $1\colon \phi_1, \ldots, 2\colon \phi_2, \ldots, 1\colon \therefore\psi$. For example, consider (45):

(45) Either it is raining or not. Suppose it is raining. Then it is wet outside. Suppose it is not raining. Then taking an umbrella would be a hassle. ??Therefore, you should take the umbrella.

This argument seems bad no matter how you parse it.

However, if 'Therefore, you should take an umbrella' is interpreted in the scope of the first supposition (that it is raining), then the argument should be fine.

Example 1 (proof by cases)

(46) Either it is raining or not. Suppose it's raining. Then better to take the umbrella. Suppose it is not raining. Then taking the umbrella will do no harm. Therefore, you should take the umbrella.

$$0\colon (r \vee \neg r); 0\colon\ + r; 1\colon \therefore u; 0\colon\ + \neg r; 2\colon \therefore u; 0\colon \therefore u$$

$$\begin{array}{l} r \vee \neg r \\ {}^{1}\left|\begin{array}{l} + r \\ \hline \therefore u \end{array}\right. \\ {}^{2}\left|\begin{array}{l} + \neg r \\ \hline \therefore u \end{array}\right. \\ \therefore u \end{array}$$

$$\left|\begin{array}{l} \text{n is prime} \\ \hline \ldots \\ \bot \end{array}\right.$$

Example 2 (nested suppositions)

(47) Alessandro is either from Turin or from Madrid. Suppose$_1$, on the one hand, that he is from Turin. Then$_1$ either he did his PhD there or he did it in the US. Suppose$_{1.1}$ he did his PhD in Turin. Then$_{1.1}$, he studied Umberto Eco's work and continental philosophy Suppose$_{1.2}$ instead he did his PhD in the US. Then$_{1.2}$ he studied analytic philosophy. Therefore$_1$, he either did continental philosophy or analytic philosophy. Now on the other hand, suppose$_2$ he is from Madrid. Then$_2$ he definitely did his PhD in the US. Therefore$_2$, he studied analytic philosophy of language. Either way, therefore, he did either continental philosophy or analytic philosophy of language.

$$\begin{array}{l} 0\colon (t \vee m), 0\colon\ + t, 1\colon \therefore (phd_t \vee phd_u), 1\colon\ + phd_t, 1.1\colon \therefore u, 1\colon\ + phd_u \\ 1.2\colon \therefore l, 1\colon \therefore (cp \vee pl), 0\colon\ + m, 2\colon \therefore phd_u, 2\colon \therefore l, 0\colon \therefore (cp \vee pl) \end{array}$$

Example 3 (Conditional Proof)

(48) Suppose the butler committed the crime. Then he left by the window. In that case, he has mud on his shoes. So if the butler committed the crime, he has mud on his shoes. [Summary uses]

$$0\colon\ + B, 1\colon \therefore W, 1\colon \therefore Ms, 0\colon \therefore B \rightarrow Ms.$$

11.1 Contexts and Labeled Trees

Think of contexts not as single information states, but rather as *labeled trees* of information states — i.e., a tree where each node is given its own label.

Idea

- The root of the tree represents the categorical information state
- The other nodes of the tree represent suppositional information states
- The "labels" keep track of which information states go with which labels in a discourse

Definitions

- An INFORMATION STATE is a set $s \subseteq W$ of worlds.
- A CONTEXT is a partial function $c\colon \mathbb{N}^{<\omega} \to \wp W$ from labels (i.e., sequences of numbers) to information states. We assume:
  (1) $0 \in \text{dom}(c)$. (The categorical state (which is the root of the tree) is always defined).
  (2) if $\langle n_1, \ldots, n_{k+1}\rangle \in \text{dom}(c)$, then $\langle n_1, \ldots, n_k\rangle \in \text{dom}(c)$. (A suppositional state is defined only when its parent state is defined. This rules out the possibility of "disconnected" segments of a branch.)
- Where n is a label, we write c_n as short for $c(n)$.
- We call c_0 the CATEGORICAL STATE of c.
- We call c_n (where $n \neq 0$) a SUPPOSITIONAL STATE of c.

Update (informal gist)

- Updating c with $n\colon \phi$ (basically) amounts to updating c_n with ϕ.
- Exception: updating with $n\colon\ +\phi$ also requires adding a new information state above c_n that's updated with ϕ. $c \oplus_n \phi$ is the result of extending c with an additional suppositional state that is copied from c_n and then updated with ϕ.

Definition 11.1 (Simple Dynamic Semantics (without the conditional)) *Where $s \subseteq W$ is an information state:*

$$\begin{aligned}
s[p] &= \{w \in s \mid w(p) = 1\} \\
s[\neg\phi] &= s - s[\phi] \\
s[\phi \wedge \psi] &= s[\phi][\psi] \\
s[\phi \vee \psi] &= s[\phi] \cup s[\psi] \\
s[\Box\phi] &= \{w \in s \mid s[\phi] = s\} \\
s[\Diamond\phi] &= \{w \in s \mid s[\phi] \neq \varnothing\}
\end{aligned}$$

Definition 11.2 (General Dynamic Semantics for Arguments) *Where ϕ does not contain $\therefore$ or $+$:*

$$c[n\colon \phi] = \begin{cases} c_n[\phi] & \textit{if } c_n \textit{ is defined} \\ \textit{undefined} & \textit{otherwise} \end{cases}$$

$$c[n\colon \therefore\phi] = \begin{cases} c[n\colon \phi] & \textit{if } c_n \textit{ is defined and } c[n\colon \phi]_n = c_n \\ \textit{undefined} & \textit{otherwise} \end{cases}$$

$$c[n\colon\ +\phi] = \begin{cases} c \oplus_n \phi & \textit{if } c_n \textit{ is defined} \\ \textit{undefined} & \textit{otherwise} \end{cases}$$

The update effect of $\therefore + \phi$ is derived from the update effects for $\therefore$ and $+$ compositionally.

$$c[n\colon \therefore + \phi] = \begin{cases} c[n\colon\ +\phi] & \text{if } c_n \text{ is defined} \\ \text{undefined} & \text{otherwise} \end{cases}$$
$$= \begin{cases} c \oplus_n \phi & \text{if } c_n \text{ is defined} \\ \text{undefined} & \text{otherwise} \end{cases}$$

Let's unpack these update clauses.

- For illustration, consider the case where $n = 0$. If ϕ does not contain $+$ or $\therefore$, then updating c with $n\colon \phi$ is the result of updating c_0, as well as any suppositional states that have been defined, with ϕ (or, more precisely, the information contained in $c_0[\phi]$). If $n \neq 0$, then the update effect is the same, except we only update information states above c_n.
- If ϕ is of the form $+\psi$, then updating c with $n\colon\ +\psi$ amounts to (i) checking whether c_n is defined, and (ii) adding a suppositional state above c_n that is the result of updating c_n with ψ. Notice that updating with $n\colon\ +\phi$ does not affect c_n: that information state is left untouched, which is precisely what we want.

  There is some question as to whether $+$ should carry an epistemic possibility presupposition — that is, whether we should require $c[n\colon \phi]_n \neq \varnothing$ in order for $c[n\colon\ +\phi]$ to be defined.

  On the one hand, there is some linguistic evidence to suggest that Suppose ϕ presupposes the epistemic possibility of ϕ.

  (49) It is not raining. ??Suppose it is raining. . .

  However, discourses containing reductio reasoning do not fit this pattern.

  For example, the following discourse sounds perfect:

  (50) There is no largest prime number. For suppose there is. . .

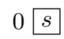
  The reductio example suggests that 'suppose' does not carry an epistemic presupposition after all. Our semantic clauses for $+$ reflects this.
- Finally, if ϕ is of the form $\therefore\psi$, then updating c with $n\colon \therefore\psi$ amounts to checking whether (i) c_n is defined, and (ii) c_n supports ψ (or, more precisely, whether the result of updating c with $n\colon \psi$ does not change the information state assigned to n). If c passes this test, then we continue to update c with $n\colon \psi$. This ensures, e.g., that $n\colon \therefore + \psi$ adds a suppositional state with the supposition ψ (as opposed to merely checking that adding that state would not crash the context, which is what would happen if the update clause simply left c in tact).

Example (proof by cases)

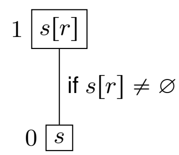
$$0\colon (r \vee \neg r), 0\colon\ +r, 1\colon \therefore u, 0\colon\ +\neg r, 2\colon \therefore u, 0\colon \therefore u$$

The effect of updating a context c with this labeled discourse is calculated as follows (throughout, let $c_0 = s$):

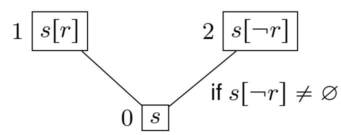
- First, we update the categorical state s with the trivial disjunction $r \vee \neg r$ (so nothing changes).



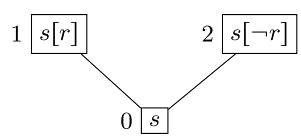
- Next, $0\colon\ +r$ requires (i) checking that $s[r] \neq \varnothing$, and if so (ii) assigning $s[r]$ to the label $0^+ = 1$ (otherwise, the resulting context is undefined).



- Then $1\colon \therefore u$ tests $s[r][u] = s[r]$. If it passes, nothing changes; otherwise, the resulting context is undefined.
- Assuming $s[r]$ passes the test, $0\colon\ +\neg r$ requires (i) checking that $s[\neg r] \neq \varnothing$, and if so (ii) assigning $s[\neg r]$ to the label $0^+ = 2$ (now that c_1 is defined).



- Then $2\colon \therefore u$ tests $s[\neg r][u] = s[\neg r]$. If it passes, nothing changes; otherwise, the resulting context is undefined.
- Assuming $s[\neg r]$ passes the test, $0\colon \therefore u$ tests $s[u] = s$. Since $s[r]$ and $s[\neg r]$ have passed this test, s will, too. So our final context is:

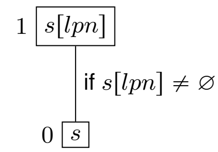


11.2 Reductio

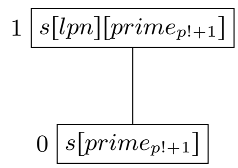
We represent (20) as:

$$0\colon\ +lpn, 1\colon \therefore composite_{p!+1}, 0\colon prime_{p!+1}, 0\colon \therefore \neg lpn.$$

- $0\colon\ +lpn$ checks that $s[lpn] \neq \varnothing$ and if so, maps 1 to $s[lpn]$.



- 1: $\therefore composite_{p!+1}$ tests whether $s[lpn]$ supports $composite_{p!+1}$.
- 0: $prime_{p!+1}$ adds $prime_{p!+1}$ to both s and $s[lpn]$.



- 0: $\therefore \neg lpn$ tests whether $s[prime_{p!+1}]$ supports $\neg lpn$. Since $s[lpn]$ supports $composite_{p!+1}$, $s[lpn][prime_{p!+1}] = \varnothing$, which trivially supports $\neg lpn$. So $s[prime_{p!+1}]$ passes the test.

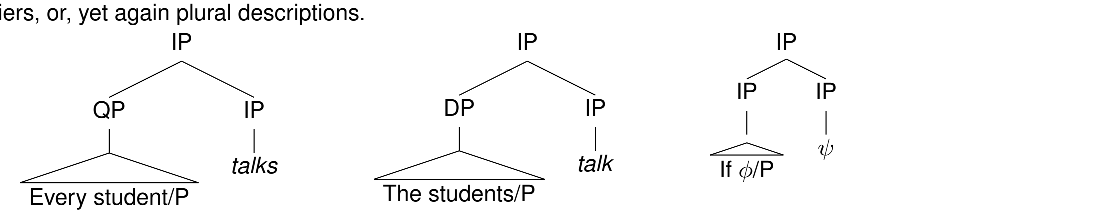
12 Conditionals, quantifiers, and plural descriptions

Definition 12.1 (Generalized Update for the Conditional (Kocurek and Pavese 2022))

$$c[n\colon \phi \to \psi] = \begin{cases} c \oplus_n \phi & \textit{if } c[n\colon \phi]_n \textit{ is defined and } c[n\colon \phi][n\colon \psi]_n = c[n\colon \phi]_n \\ c \Uparrow_n \varnothing & \textit{otherwise} \end{cases}$$

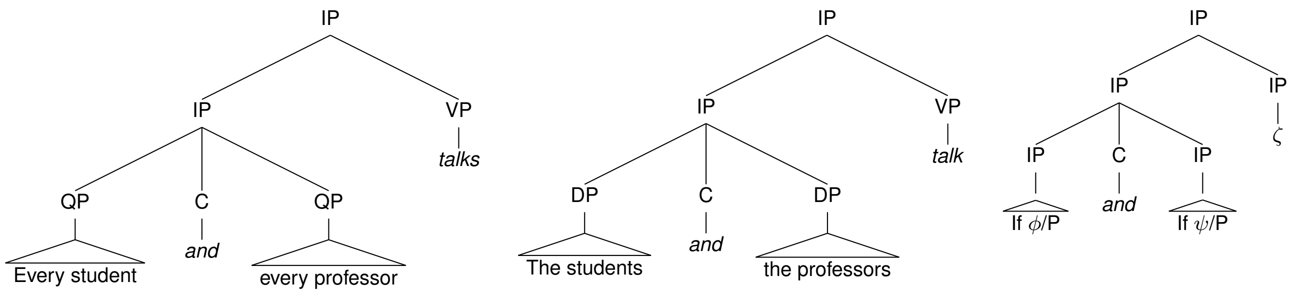
But this obliterates the differences between arguments and conditionals. So this has to be revised.

A long tradition in semantics takes if-clauses to quantify over possible worlds, or be restrictors of hidden quantifiers, or, yet again plural descriptions.



Another observation in favor of this view is **Observation** $\#1$. Coordinating 'ifs' is not surprising if 'if'-clauses are quantifiers or plural descriptions, since quantifier phrases and plural descriptions coordinate too:

(51) a. ✓ Every student and every professor talks.
b. ✓ The students and the professors talk.
c. ✓ If the dice comes up 4 and if it comes up 2, Ben will win.
d. # Suppose the dice comes up 4 and suppose it comes up 2. Then Ben will win.



13 Different Mechanisms of Modal Subordination

13.1 Arguments

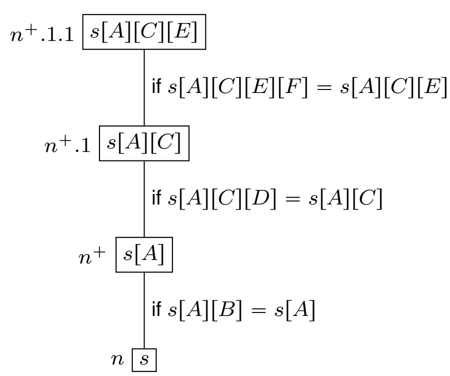
Modal subordination is due to the fact that the test operated by arguments returns not just the original context prior to the update with the supposition but a new node with a new suppositional state updated with the supposition. Further subordinated suppositions are then interpreted relative to that new context.

In general, one discourse is modally subordinated to another if the former has a label that was introduced by the latter. An important element of the framework is that which labeled sentences are modally subordinated to which labeled sentences is represented by the labels: either the subordinated argument have the same label or they have an incremental label.

Persistence

(52) Suppose a wolf comes in. We will use a gun. Suppose we manage to shoot. In that case, we will be safe. Suppose we bury the body. In that case, nobody will find out.

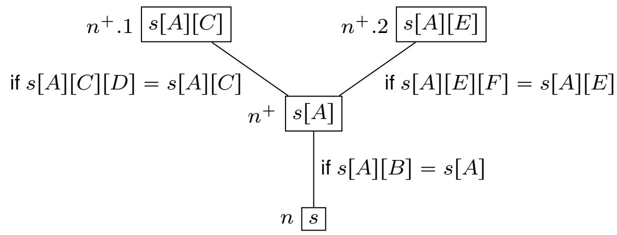
$$n\colon\ +A, n^{+}\colon \therefore B, n^{+}\colon\ +C, (n^{+}).1\colon \therefore D, (n^{+}).1\colon\ +E, (n^{+}).1.1\colon \therefore F$$



Reversibility

(53) Suppose a wolf comes in. We will use a gun. Suppose we manage to shoot. We will be safe. Suppose we do not manage to shoot it. We will be in trouble.

$$n\colon\ +A, n^{+}\colon \therefore B, n^{+}\colon\ +C, (n^{+}).1\colon \therefore D, n^{+}\colon\ +D, (n^{+}).2\colon \therefore F.$$



Conditional The modal subordination is modeled for by letting the antecedent of conditionals involve an anaphoric element, which refers to a salient proposition, which then is computed in the update of the context with the conditional. The antecedent of a conditional does not introduce a new node. When the pattern of modal subordination is reversed, it is because the if-clause is let anaphorically refer to a different set of possibilities.