On the Modeling of Live Possibilities

Consider the following exchange adapted from Dretske [3]:

(1) John: There is only one zebra in the pen today.
Mary: It might be a zebra, but it also might be a cleverly disguised mule.
John: Oh, I know their budget was tight but I didn't know it was this bad.

On an intuitive understanding of the scenario, John was unaware of the possibility of the zebra being a cleverly disguised mule before Mary mentioned it. While this possibility is not ruled out by what he knows, it is not a salient possibility that he was attending to. The effect of Mary's *might*-claim can be understood as making this plain possibility live to John.

Despite the intuitiveness of the notion, it remains unclear how to best cash out live possibilities formally. To begin, if we construe information states (notated by s) simply as sets of possible worlds, then it is impossible to distinguish live from plain possibilities. One natural maneuver is then to add more structure by moving one level up on the set-theoretic hierarchy and represent bodies of information using sets of sets of possible worlds; call them *hyperstates* (notated by S). At the level of hyperstates, p is viewed as a plain possibility iff S contains a p–world; as for live possibilities, there are now two distinct ways to define them: a supervaluation-based analysis [6, 7], and a alternative-based on analysis [2, 4, 1].

S(upervaluation)-Based: p is a live possibility in S iff $\forall s \in S.\exists w \in s : w \in V(p)$.

A(lternative)-Based: p is a live possibility in S iff $\exists s \in S.\forall w \in s : w \in V(p)$.

This paper offers a critical comparison of the two approaches. To do so, we first delineate several desiderata that we believe an ideal analysis of live possibilities should satisfy. We argue that none of the existing approaches, be it S-based or A-based, fulfils all of them. We then sketch a A-based account that manages to capture all the desiderata.

1 Desiderata

We first define a notion of support of a formula by a body of information as employed in dynamic semantics (cf., [5]): S supports φ ($S \vDash \varphi$) iff $S[\varphi] = S$, where the latter means that updating S with $[\varphi]$ returns S unchanged. We postulate the following list of desiderata:

Closure under Negation(CN): $S \vDash \Diamond p$ but $S \nvDash p$, then $S \vDash \Diamond\neg p$.

Non Closure under Conjunction(NCC): $S \vDash \Diamond p \wedge \Diamond q$ does not entail $S \vDash \Diamond(p \wedge q)$.

Decomposable under Conjunction(DC): $S \vDash \Diamond(p \wedge q)$ entails $S \vDash \Diamond p$ and $S \vDash \Diamond q$.

Dynamic Attentiveness(DA): If $S \vDash \Diamond p$ and $S[\varphi] = S'$, then if $S' \nvDash \neg p$, $S' \vDash \Diamond p$.

CN says that if p is a live possibility but has yet to be established as true, then $\neg p$ should also be a live possibility. This constraint seems intuitively plausible: anyone who actively entertains p but falls short of accepting p should also consider $\neg p$ as a live option. To illustrate, consider (2), which sounds odd as it violates **CN**:

(2) #I don't know whether Alice will come to the party. She might come but it's not the case that she might not come.

Similarly, **NCC** and **DC** can be illustrated by the contrast between (3) and (4):

(3) Alice might come to the party. Bob might come as well. But they won't both come.

(4) #Alice and Bob might both come to the party, but Alice won't come.

If live possibilities were closed under conjunction, we would expect the last sentence in (2) to contradict the conjunctive possibility. By contrast, (4), unlike (3), does sound contradictory. Lastly, **DA** states that if p is a live possibility in S, then as long as updating S does not establish the truth of $\neg p$, p should remain as a live possibility. In other words, once a possibility has been raised to salience, it will stay salient unless it has been ruled out.

2 Evaluating Existing Accounts

We now examine how the existing accounts fare with the above desiderata. Consider the **S-based** approach [6, 7], under which p is a live possibility just in case it is compatible with every relevant body of information $s \in S$ in inquiry. This account successfully captures **NCC** – some s can contain both a p-world and a q-world but not any $(p \wedge q)$-world – and **DC** – any s that contains a $(p \wedge q)$-world must contain both a p-world and a q-world. However, it fails to capture both **CN** and **DA**. While **CN** can be recaptured by requiring any update with $[\Diamond p]$ be accompanied by an update with $[\Diamond \neg p]$, there does not seem to be an easy fix for **DA**. To give the gist of the problem, the update with $[q]$ on S will eliminate all $\neg q$-worlds from every $s \in S$. Suppose $S \vDash \Diamond p$; then updating with $[q]$ may inadvertently eliminate some $(p \wedge \neg q)$-world so that some of the resulting s no longer contain any p-world, thereby demoting p from a live possibility to a plain possibility. (See Example 1 for a concrete case.)

On **A-based** accounts ([2, 4, 1]), p is a live possibility just in case p corresponds to a salient alternative in S. That is, S contains an s that consists of all and only the p-worlds in S. As such, **DA** is ensured: since eliminating q-worlds from S will only shrink the size of each alternative; so long as p is still possible in S, it will remain live. However, the existing **A-based** accounts fail to jointly satisfy **CN** and **NCC**. Our account aims to address this.

3 A New A-Based Account of Live Possibilities

We present an update semantics that divides an update $S[\varphi]$ into two steps: an additive update $[\varphi]_a$ which adds new alternatives to S thereby bringing certain possibilities to salience, and then a traditional eliminate update $[\varphi]_e$ which eliminates worlds incompatible with φ.

Consider the additive update first: we define $S[\varphi]_a$ as $S \cup att(\varphi)$, where (to borrow a term from [4]) a formula's attentive content $att(\varphi)$ is defined recursively as follows:

- (i) $att(p) := \{|p|, |\neg p|\}$, where $|p|$ (or $|\neg p|$) is the set of worlds where p is true (or false);
- (ii) $att(\neg\varphi) := att(\varphi)$; (iii) $att(\varphi \wedge \psi) := att(\varphi) \cup att(\psi)$; (iv) $att(\varphi \vee \psi) := att(\varphi) \cup att(\psi)$

Note that (i) says that drawing attention to p also brings $\neg p$ to salience; this in turn secures **CN**. The attentive content of a Boolean formula is simply the sum of the attentive content of its parts. Now, to capture the difference between the updates $[\Diamond(p \wedge q)]_a$ and $[\Diamond p \wedge \Diamond q]_a$, we construe the attentive content associated with the former as consisting of the joint possibility $(p \wedge q)$ as well as the possibilities evoked by its negation, namely $\neg p$ and $\neg q$. The negative possibilities are so decided as to ensure the joint satisfaction of **DC** and **CN**: given **DC** drawing attention to $p \wedge q$ also draws attention to p and to q, which given **CN** also brings $\neg p$ and $\neg q$ to salience. Thus, we define $att(\Diamond\varphi)$ as conjoining the positive and negative possibilities evoked by φ, namely as $||\varphi|| \cup ||\neg\varphi||$, where $||\varphi||$ is recursively defined as follows:

- (i) $||p|| := \{|p|\}$; (ii) $||\neg p|| := \{|\neg p|\}$; (iii) $||\varphi \wedge \psi|| := \{x \cap y | x \in ||\varphi|| \ \& \ y \in ||\psi||\}$;
- (iv) $||\neg(\varphi \wedge \psi)|| := ||\neg\varphi \vee \neg\psi||$; (v) $||\varphi \vee \psi|| := ||\varphi|| \cup ||\psi||$; (vi) $||\neg(\varphi \vee \psi)|| := ||\neg\varphi \wedge \neg\psi||$.

As for the eliminative updates, updating S with $[\varphi]_e$ amounts to updating every $s \in S$ with $[\varphi]_e$ according to the standard Veltman [5] update clauses and subsequently collecting all the non-empty output sets. The only exception is when we update s with $[\Diamond\varphi]_e$. This update returns s if φ is already live in S, i.e., when $\exists s' \in S : s'[\varphi] = s'$; otherwise $s[\Diamond\varphi]_e = \varnothing$. Therefore, the eliminative update $S[\Diamond\varphi]_e$ will always return either S or the absurd state $\varnothing$.

References

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Example 1

To illustrate the failure of **DA** under a strengthened S-based account that satisfies the rest of our desiderata, consider updating the initial hyperstate S_0 where no possibilities are live with $[\Diamond p \wedge \Diamond q]$ and then with $[q]$. Given a set of world W, we can define the initial hyperstate S_0 as the power set of W excluding the set of the empty set: $\wp(W) - \{\varnothing\}$. Let pq be a $(p \wedge q)$-world, $p\bar{q}$ be a $(p \wedge \neg q)$-world, and so on. The updates then proceed as follows:

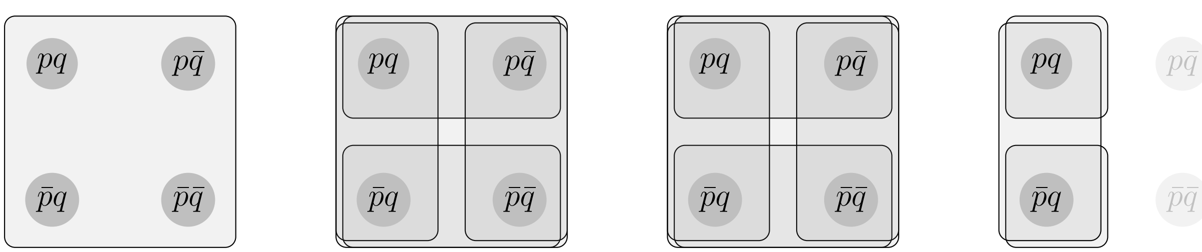
1. Updating with $[\Diamond p \wedge \Diamond q]$ makes both p and q live possibilities, but given **CN**, it also makes $\neg p$ and $\neg q$ live possibilities. Hence, the output hyperstate S_1 is a set that consists of the following sets of worlds: $\{pq, \bar{p}\bar{q}\}, \{p\bar{q}, \bar{p}q\}, \{pq, p\bar{q}, \bar{p}q\}, \{pq, p\bar{q}, \bar{p}\bar{q}\}, \{pq, \bar{p}q, \bar{p}\bar{q}\}, \{p\bar{q}, \bar{p}q, \bar{p}\bar{q}\}$, and $\{pq, p\bar{q}, \bar{p}q, \bar{p}\bar{q}\}$.

2. Updating with $[q]$ then eliminates all the $\bar{q}$-worlds from every $s \in S_1$. The output hyperstate S_2 is a set that consists of: $\{pq\}$, $\{\bar{p}q\}$, and $\{pq, \bar{p}q\}$.

But since not every $s \in S_2$ contains a p-world, p is no longer a live possibility; it has changed from a live possibility to a merely plain possibility. This means **DA** fails.

Example 2

The following diagram illustrates the same sequential update first with $[\Diamond p \wedge \Diamond q]$ and then with $[q]$ on our account. The initial hyperstate S_0 is defined as $\{W\}$, and each update is divided into an additive update $[\ ]_a$ and an eliminative update $[\ ]_e$.



(a) S_0 (b) $S_0[\Diamond p \wedge \Diamond q]_a = S_1$ (c) $S_1[\Diamond p \wedge \Diamond q]_e[q]_a = S_1$ (d) $S_1[q]_e = S_2$

Since the final output hyperstate S_2 contains a p-alternative, namely the set $\{pq\}$, p-remains as a live possibility. Thus, we capture **DA**. Additionally, note that in S_1, both p and q, as well as their negations, are live possibilities whereas their conjunction is not. Hence, this example also illustrates how **CN** and **NCC** are satisfied on our account.