Trivalent Exh and the Exclusion theory of summative predicates

Summary: The exhaustivity-based Exclusion theory of summative predicates faces three difficulties that are overcome by adopting the trivalent exhaustivity operator of Bassi et al. (2021).

Background: the Exclusion theory of summative predicates. Predicates are 'summative' if they are true of an individual by virtue of being true of that individual's parts (e.g. colour terms, material terms). While summative predicates are true of *all* parts of their argument in positive sentences, they are true of *no* parts of their argument in negative sentences (Löbner 2000):

(1) a. The square is blue.
$\approx$ all of the square is blue
$\not\approx$ at least some of the square is blue

b. The square isn't blue.
$\not\approx$ not all of the square is blue
$\approx$ none of the square is blue

How should this quantification be captured? Harnish (1976), Levinson (1983), and Paillé (2021) suggest that summative predicates are lexically existential (2). (1b) follows immediately. For (1a), they suggest that colours *exclude* other colour terms through exhaustification (3).

(2) $[\![\text{blue}]\!] = \lambda x.\exists y[y \sqsubseteq x \wedge \mathsf{blue}(y)]$. (abbreviated as '$\lambda x.\mathsf{blue}_{\exists}(x)$')

(3) a. Exh_{ALT} [The square is blue].
b. ALT = {The square is blue$_{\exists}$, The square is white$_{\exists}$, The square is red$_{\exists}$, ...}
c. $[\![(3a)]\!] = 1$ iff $\mathsf{blue}_{\exists}(s) \wedge \neg\mathsf{white}_{\exists}(s) \wedge \neg\mathsf{red}_{\exists}(s) \wedge \neg \ldots$

If the square is at least partly blue and has no other colour, it must be entirely blue.

We now turn to seeing that, as it stands, this theory fails to capture central facts about summative predicates. These issues are solved by adopting the trivalent Exh of Bassi et al. (2021).

Problem 1: Truth-value gaps. The falsity conditions of summative predicates are not the complement of their truth conditions (1) (Löbner 2000; Križ 2015). Rather, if the subject is neither all blue nor not blue at all, positive/negative sentences are undefined (4). Call this a 'heterogeneity-gap.'

(4) $$[\![\text{The square is blue}]\!] = \begin{cases} 1, \text{ if the square is all blue;} \\ 0, \text{ if the square is not blue at all;} \\ \#, \text{ otherwise} \end{cases}$$

The Exclusion account does not predict a heterogeneity-gap, but there is a simple way to fix this.

The above discussion tacitly assumed the standard Exh operator of Chierchia et al. (2012) and related work (5). In (5), IE(ϕ) is the set of ϕ's innocently excludable alternatives (Fox 2007).

(5) $[\![\text{Exh}(\phi)]\!] = 1$ iff $[\![\phi]\!] = 1 \wedge \bigwedge([\![\psi]\!] = 0 : \psi \in \text{IE}(\phi))$

In contrast, Bassi et al. (2021) propose a *trivalent* exhaustivity operator ('Pexh'), which excludes alternatives in the truth conditions, but has falsity conditions based on only the prejacent:

(6) $$[\![\text{Pexh}(\phi)]\!] = \begin{cases} 1, \text{ if } [\![\phi]\!] = 1 \wedge \bigwedge([\![\psi]\!] = 0 : \psi \in \text{IE}(\phi)); \\ 0, \text{ if } [\![\phi]\!] = 0; \\ \#, \text{ otherwise} \end{cases}$$

By replacing Exh with Pexh, the Exclusion theory obtains heterogeneity-gaps, as desired:

(7) $$[\![\text{Pexh}_{\text{ALT}} \text{ [the square is blue]}]\!] = \begin{cases} 1, \text{ if } \mathsf{blue}_{\exists}(s) \wedge \neg\mathsf{white}_{\exists}(s) \wedge \neg\mathsf{red}_{\exists}(s) \wedge \neg \ldots; \\ 0, \text{ if } \neg\mathsf{blue}_{\exists}(s); \\ \#, \text{ otherwise} \end{cases}$$

Indeed, consider the logical value obtained if the square is half blue, half white. Here, it is neither the case that $\neg\mathsf{white}_{\exists}(s)$ holds (as needed for the sentence to be true), nor that $\neg\mathsf{blue}_{\exists}(s)$ holds (as needed for the sentence to be false). Hence, the sentence is undefined.

Problem 2: Non-maximality. In plural predication, 'non-maximality' refers to the ability of positive sentences to tolerate exceptions; e.g., (8a) can be felicitous even if not quite all the professors smiled (Križ 2015). Non-maximality is observed with summative predicates too: (8c) can be felicitous even if the table has some minor wooden parts, for instance. Non-maximality disappears with *all* (8b,d), as do heterogeneity-gaps, suggesting a connection between the two.

(8) a. The professors smiled.
b. All the professors smiled.
c. The table is metal.
d. All of the table is metal.

For plurals (8a), Križ (2015:76ff) assumes that worlds in conversation are partitioned according to how they resolve a QUD, and suggests that the maxim of Quality is weak: speakers must only say things that are *true enough*, i.e. in the same cell as a literally true sentence. This allows speakers to say things that are *undefined*, but not *false*—a sentence *S* does not felicitously address an issue if there is a cell where *S* is true in some worlds but false in others. Thus, QUD permitting, (8a) can be used if only most professors smiled (being undefined), but not (8b) (being false).

Deriving the meaning of (8c) through the classic Exh makes the sentence *false* if the table is not exclusively metal. But with Pexh, undefinedness obtains for a mixed-material table. Križ's mechanism can therefore work as with plurals, predicting the possibility of non-maximality.

Problem 3: Negation vs. other DE contexts. An odd fact about summative predicates (cf. Križ 2015 on plurals) is that, while they are weak under *not*, they remain strong in other DE contexts:

(9) If the square is blue, do five jumping-jacks. (≈ if the square is entirely blue, ...)

On the Exclusion account, this means stipulating a locality constraint (which one hopes to eventually derive); Exh must appear below *if* in (9). Such a constraint is not specific to DE contexts; in (10), for instance, Exh must be below *every* for *blue* to be strong.

(10) Every square is blue. (≈ every square is entirely blue)

If Exh is bivalent, there would have be an exception to this locality constraint; after all, an Exh under *not* in (1b) would create the wrong meaning ('it is not the case that the square is only blue'). It isn't clear how a theory of Exh's locality could distinguish between negation and other DE operators. But with Pexh, the problem disappears. The locality constraint can be claimed to be absolute, with Pexh vacuously appearing below negation:

(11) $$[\![\text{not [Pexh}_{\text{ALT}}\text{ [the square is blue]]}]\!] = \begin{cases} 1, \text{ if } \neg\mathsf{blue}_{\exists}(s); \\ 0, \text{ if } \mathsf{blue}_{\exists}(s) \wedge \neg\mathsf{red}_{\exists}(s) \wedge \neg\mathsf{white}_{\exists}(s) \wedge \neg \ldots; \\ \#, \text{ otherwise} \end{cases}$$

Pexh captures that negation behaves differently from other DE environments. (11) also makes it possible to derive non-maximality in negative sentences in the same way as in positive ones (this benefit of Pexh carries over to Bar-Lev's (2021) exhaustivity-based theory of plural predication).

Conclusion. We have seen some shortcomings of the Exclusion theory of summative predicates. These shortcomings can all be overcome by replacing Exh with Pexh. This constitutes both an improvement of the Exclusion theory, and—if exhaustification is the right way to explain the strength of summative predicates—an argument in favour of Pexh.

References

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