Introduction: Partee disjunctions such as (1) have been subject to a recent surge of interest.
(1) Either Giles doesn't own $\mathbf{a}^{x}$ donkey, or he treasures $\mathbf{i t}_{x}$.

Descriptively, these are sentences of the form $P$ or $Q$, where (i) $P$ contains an existential statement $\phi$ (ii) and the negation of $P$ contextually-entails a witness to $\phi ; Q$ contains a discourse-anaphoric pronoun. It's well-known that 'first-generation' dynamic semantics (Heim 1982, Groenendijk \& Stokhof 1991) can't capture (1). Subsequent theories have accounted for Partee disjunctions by rethinking how negation and disjunction interact with anaphora. Approaches differ however in the truth-conditions they attribute to Partee disjunctions. Krahmer \& Muskens (1995) claim that (1) has a strong $\forall$-reading, as in (2). Elliott's $(2020,2022)$ bilateral dynamic semantics, on the other hand, predicts a weak existential reading, as in (3). Mandelkern 2022, Hofmann 2022 don't explicitly discuss the issue, but also predict an $\exists$-reading. Concretely, imagine that Giles has two donkeys; one he treasures, and one he doesn't - on the $\forall$-reading (1) is true, and on the $\exists$-reading (1) is false.
(2) Giles treasures every donkey that he owns
(3) If Giles owns a donkey, then Giles treasures a donkey that he owns

The empirical contribution of this paper is to argue that (2) and (3) each cover only part of the range of attested readings. Like donkey sentences, Partee disjunctions oscillate between $\forall$ - and $\exists$-readings, in a way which is sensitive to contextual factors. The theoretical contribution is to develop a more general account of $\forall / \exists$-readings extending beyond determiners, inspired by Champollion, Bumford \& Henderson 2019 (henceforth: CBH).
$\forall / \exists$-readings of Partee disjunctions: We'll begin by motivating the $\exists$-reading. Consider the following context: Did Gennaro pay for dinner with cash or card. He sometimes forgets his wallet, which has multiple credit cards inside. (4) is clearly true in scenario where Gennaro paid with one of his multiple credit cards. Partee disjunctions in such mixed scenarios are only captured by the $\exists$-reading.
(4) Either he doesn't have $\mathbf{a}^{x}$ credit card with him, or he paid with $\mathbf{i t}_{x}$.

Now, let's motivate the $\forall$-reading. Consider the following context: plumbers $M$ (ario) and L(uigi) are inspecting two storey houses to determine if they need a downstairs bathroom installed. M can get a sense of a house's plumbing just by looking at its exterior. L asks does this house need a downstairs bathroom installed?; $M$ responds with (5). In this context, $M$ clearly conveys that every bathroom in this house is upstairs; he's wrong in the mixed scenario where there's also bathroom downstairs. This is incompatible with the $\exists$-paraphrase in (3), but expected according to the $\forall$-paraphrase.
(5) Yes - Either there's no ${ }^{x}$ bathroom in this house, or it ${ }_{x}$ 's upstairs.

Embedding a Partee disjunction in a negative context displays a clear bias for a strong (negated) $\exists$ reading. Consider: I ask my students to submit one or more questions three days before class. Shane has a reputation for being unreliable, but my teaching assistant reports to (6). Intuitively, this is true iff Shane submitted all of his questions on time; the TA reported something incorrect in a mixed scenario where Shane submitted one of his questions late.
(6) Neither did Shane submit no ${ }^{x}$ question, nor did he submit $\mathbf{i t}_{x}$ late.

Analysis: We adopt Elliott's (2022) Bilateral Update Semantics (BUS) as an account of Partee disjunctions, according to which a sentence $\phi$ is associated with both positive and negative updates on Heimian information states $s[\phi]^{+}, s[\phi]^{-}$. BUS has the virtue of (a) following from a general strategy for lifting Strong Kleene connectives into a dynamic setting, and (b) drawing a tight connection between anaphora and presupposition projection. Given a schematic Partee disjunction $\neg \exists_{x} B(x) \vee U(x)$, (i) the positive update (7) returns possibilities from $s$ where there is no bathroom, and (ii) possibilities from $s$ indeterministically extended with a bathroom upstairs discourse referent; possibilities in which
there is no bathroom upstairs are eliminated. The negative update (8) returns possibilities from $s$ indeterministically extended with a bathroom not-upstairs discourse referent. Note importantly that the output of both updates may contain mixed possibilities, in which there is a bathroom upstairs, and a bathroom not upstairs.

$$
\begin{equation*}
s\left[\neg \exists_{x} B(x) \vee U(x)\right]^{+}=\left\{(w, g) \in s \mid B_{w}=\varnothing\right\} \cup\left\{(w, h) \mid(w, g) \in s, g[x] h, h_{x} \in B_{w}, U_{w}\right\} \tag{7}
\end{equation*}
$$

(8) $s\left[\neg \exists_{x} B(x) \vee U(x)\right]^{-}=\left\{(w, h) \mid(w, g) \in s, g[x] h, h_{x} \in B, \notin U_{w}\right\}$

Elliott suggests that the effect of assertion in BUS amounts to the positive update. An additional condition demands that every world in the input state ends up in the positive or negative update, to capture Heimian familiarity using partial assignments. This is stated in (9). For Partee disjunctions, this predicts that mixed scenarios aren't excluded, and therefore the $\exists$-reading is derived.
(9) Bridge (v1): $\phi$ asserted at $c$ results in $c[\phi]^{+}$if $\forall(w, *) \in c, w \in c[\phi]^{+}$or $w \in c[\phi]^{-}$else $\varnothing$ Surprisingly, (8) predicts that negated Partee disjunctions receive weak, (negated) $\forall$-readings, since mixed scenarios aren't excluded here either. BUS therefore has a rather surprising property - even if a Partee disjunction $\phi$ is bivalent, the information conveyed by $\phi$ overlaps with the information conveyed by $\neg \phi$. The core idea behind our analysis is that there is a preference for interpreting $\phi$ and $\neg \phi$ as inducing a contextual partition by excluding possibilities at which $\neg \phi$ is true when $\phi$ is asserted. This is formalized in the modified bridge principle in (10).
(10) Bridge (v2): $\phi$ asserted at $c$ results in $\left\{(w, g) \in c[\phi]^{+} \mid(w, *) \notin c[\phi]^{-}\right\}$

$$
\text { if } \forall(w, *) \in c, w \in c[\phi]^{+} \text {or } w \in c[\phi]^{-} \text {else } \varnothing
$$

A natural worry is that (10) will over-strengthen simple existential statements, erroneously predicting unattested $\forall$-readings. This is unfounded, since existential statements in BUS semantically induce a contextual partition: $\neg \exists_{x} P(x)$ is false relative to a mixed scenario, as indicated in (12).

$$
\begin{equation*}
s\left[\exists_{x} P(x)\right]^{+}=\left\{(w, h) \mid(w, g) \in s, g[x] h, h_{x} \in P_{w}\right\} \tag{11}
\end{equation*}
$$

(12) $\quad S\left[\exists_{x} P(x)\right]^{-}=\left\{(w, g) \in s \mid P_{w}=\varnothing\right\}$

Our account is highly reminiscent of CBH's trivalent account, where instead of overlap, mixed scenarios result in undefinedness. Unlike CBH however, we don't require a distinct bridge principle for open sentences such as "it ${ }_{x}$ 's upstairs", which can also result in undefinedness if $x$ isn't contextually familiar. This is because we distinguish between overlapping updates and partiality. The role of context: We still need a way of deriving weak readings, as on Elliott's original account. To do so, we follow CBH in assuming that sentences are interpreted relative to a salient QUD, represented as an equivalence relation $\sim_{Q}$, which induces a partition on the context set. The final bridge principle has us retain worlds in the positive update if there is some $Q$-equivalent world that isn't in the negative update.
(13) Bridge (final): $\phi$ asserted at $c$ results in $\left\{(w, g) \in c[\phi]^{+} \mid \exists w^{\prime}, w \sim_{Q} w^{\prime},\left(w^{\prime}, *\right) \notin c[\phi]^{-}\right\}$

$$
\text { if } \forall(w, *) \in c, w \in c[\phi]^{+} \text {or } w \in c[\phi]^{-} \text {else } \varnothing
$$

Just like CBH on donkey sentences, we predict that Partee disjunctions always receive strong (either universal, or negated existential) readings relative to a default "fact finding" $Q$, where no two worlds are $Q$-equivalent. Weak readings obtain when Partee disjunctions are interpreted relative to questions which nullify the difference between mixed and non-mixed scenarios. E.g., if the question is Did Gennaro pay with a credit card?, we predict a weak reading for (4), since mixed worlds (i.e., ones in which Gennaro paid with one of his credit cards, but has others he didn't pay with), will be $Q$ equivalent to worlds in which Gennaro paid with all of his credit cards. In the talk, we show how overlapping updates subsumes CBH's account, by stating a general dynamic schema for determiners. (Diagrams illustrating the basic system can be found on the following page.)

Figure 1: Dynamics of simple sentences. exhaustively indicate which individuals are $P$. The blue region is the positive update, and the red region the negative update

Figure 2: Dynamics of existential statements. Subscripts on worlds exhaustively indicate the individuals that are $P$.


Figure 3: Dynamics of Partee disjunctions. Subscripts iconically indicate which storey bathrooms $a, b$ are on. The $\frac{a}{b}$ and $\frac{b}{a}$ worlds is in both the positive and negative update.


Figure 4: Effect of asserting a Partee disjunction in a fact-finding context


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